MHD COUETTE FLOW IN AN INCLINED VERTICAL CHANNEL WITH HEAT SOURCE

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ABSTRACT
This paper investigates the problem of MHD Couette flow in an incline vertical channel with a heat source. The ordinary differential equations governing the flow are solved analytically; the result shows that there is a decrease with an increase in Grashof number (Gr) and also a decrease with an increase in heat source (Q) and Magnetic field (M) on velocity. And lastly, the temperature decreases with an increase in the heat source.

Keywords: Velocity, Temperature, Grashof number, Heat source and Magnetic field

1. INTRODUCTION

Magneto-hydro-dynamic (MHD) is the knowledge of the motion of an electrically conducting fluid like molten iron, mercury, and plasma due to a magnetic field. MHD flow has seen an extensive range of applications in the modern past and has gained significant interest owing to those in geophysical and cosmic fluid dynamics. Many experimental, mathematical and hypothetical studies of the free convection flow of viscous fluids between two plates in vertical channels and a cylinder under an inclined magnetic field have been reported in the literature—for example, Singha and Deka (2009). We investigated the two-phase MHD flow in a consistently inclined magnetic field. We obtained the analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. Unsteady MHD Couette flow bounded between two parallel porous plates with heat transfer and an inclined magnetic field was considered by Daniel *et al.* (2014).

The study of unsteady Magneto-hydro-dynamic (MHD) Couette flow between two parallel horizontal porous plates in an Inclined Magnetic Field has many applications in different engineering and technology fields. The interaction between the conduction fluid and the magnetic field radically modifies the flow, with effects on such important flow properties as heat transfer, the detail nature of which strongly depends on the orientation of the magnetic field. When the fluid moves through a magnetic field, an electric field or, consequently, a current may be induced, and in turn, the current interacts with the magnetic field to produce a body force on the fluid. The production of this current has led to MHD power generators, MHD devices, and nuclear engineering, and the possibility of thermonuclear power has created a significant practical need for understanding the dynamics of conducting. The influence of a magnetic field in the viscous incompressible flow of electrically conducting fluid is of use in the extrusion of plastics in manufacturing rayon, nylon etc.

However, the Dufour effect on unsteady free convection flows in a vertical channel under an inclined magnetic field, heat generation or absorption. The first-order chemical reaction has yet to be addressed in the literature, even though this problem finds potential applications in chemical processing equipment, geothermal energy systems, cooling of nuclear reactors and the design of heat exchangers. There is not much information on the Dufour effect compared to the Soret effect for double-diffusive natural convection flows.

Moreover, Eckert (1972). highlighted several cases of the Dufour effect's influence on the convective heat and mass transfer process. Also, studying fluid flow through porous media and heat transfer is fundamental. Reddy (2014). Discussed the MHD Couette flow of incompressible viscous fluid through a porous medium between two parallel plates under the influence of an inclined magnetic field. The effect of mass and heat transfer on unsteady MHD Poiseuille flow between two parallel porous plates in the presence of an inclined magnetic field is studied by Joseph *et al.* (2015). Unsteady MHD Poiseuille flow with heat and mass transfer between two infinite parallel plates through a porous medium in an inclined magnetic field was investigated by Kumar (2015).

The influence of the inclined magnetic field and variable thermal conductivity on MHD plane Poiseuille flow past non-uniform plate temperature was considered by Mburu et al. (2016), who discussed the MHD fluid flow between two infinite parallel plates subjected to an inclined magnetic field and pressure gradient. Nyariki *et al.* (2017) analyzed the unsteady Hydro magnetic Couette flow in a variable inclined magnetic field. The unsteady magnetohydrodynamic flow of viscous, electrically conducting fluid bounded between two nonconducting vertical plates with an inclined magnetic field was studied by Goswami and Singha (2018). Heat and mass transfer on unsteady MHD flow in two nonconducting infinite vertical plates with inclined magnetic fields is investigated by Hemamalini (2018). In the above studies, the effects of Sorat and Dufour are neglected in heat and mass transfer processes by assuming that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws.

In a different paper, Hamza *et al.* (2021) investigated the implications of MHD on the free convective slip flow of an exothermic fluid under the influence of Newtonian heating. Taid and Ahmed (2022) used the perturbation approach to resolve the sort effect, heat dissipation, and chemical reaction effects on steady two-dimensional hydro-magnetic free convection flows across an inclined porous plate coated with porous media. Osman *et al.* (2022) outlined magneto-hydrodynamics action on free convection flow over an infinite inclined plate using the Laplace transformation technique. Siva *et al.* (2021) presented a precise response to the MHD action in a heat transfer study of electroosmotic flow in a rotating microfluidic channel.

This paper investigates the problem of MHD Couette flow in an incline vertical channel with a heat source. The ordinary differential equations governing the flow are solved analytically. Analytical solutions for velocity, temperature, and concentration are obtained for suitable boundary conditions.

2. PRELIMINARIES

This section gives some applicable definitions and preliminaries in our work.

2.1 Magneto-hydrodynamic(MHD)
is the study of the magnetic properties of electrically conducting fluids, such as plasmas, liquid metals, salt water, and electrolytes?

2.2 Fluid
This article is about the concept in physics for other uses. See fluid (disambiguation). Not to be composed of liquid. In physics, a fluid is a substance that contains deformation (flow) under applied shear stress. Fluids are a subset of the phases of matter and include gas, liquid plasma, and, to some extent, plastic solids. Fluids are substances that have zero shear force applied to them.

2.3 Vertical Channel
refers to a physical transmission medium such as wire or logical convectional over a multi-mixed medium such as a radio channel. A channel with certain capacity transmission information is often measured by its hand within Hz or its data rate in bit per second.

2.4 Flow
When material moves freely from one place or from one place to another, large numbers or amounts in a steady, unbroken street.

2.5 Boundary layer
Is the layer of the fluid near a boundary surface when the effects of viscosity are significant? When a fluid rotates, viscous forces are balanced by the carioles effect (rather than convective interim).

2.6 Exact Solution:
Refers to a solution that captures the entire physical and mathematics of a problem as composed to one that is approximate perturbative, etc., the exact solution; therefore, it does not need to be closed form.

3.1 METHODOLOGY

3.1 MHD Couette flow in an inclined vertical channel with a heat source
The equation governing the flow problem of MHD Couette flow in an inclined vertical channel with a heat source.

$$\frac{\partial^2 u}{\partial y^2} - M = -Gr\sin(\alpha)\theta \quad (3.1.1)$$

$$\frac{\partial^2\theta}{\partial y^2}+Q\theta=0 \tag{3.1.2}$$

$$\left.\begin{array}{llll} U=0, & \theta=1 & at & y=0 \\ U=1 & \theta=0 & at & y=1 \end{array}\right\} \tag{3.1.3}$$

Solution

From equation (3.1.2)

$$\frac{\partial^2\theta}{\partial y^2}+Q\theta=0$$

$$M^2+Q=0$$

$$M=\sqrt{-Q}$$

$$M=\pm i\sqrt{Q}$$

$$\theta=A_1\cosh\sqrt{Qy}+A_2\sinh\sqrt{Qy} \tag{3.1.4}$$

Apply boundary conditions

$$At \quad \theta=1 \quad y=0$$

$$1=A_1\cosh\sqrt{Q(0)}+A_2\sin\sqrt{Q}(0)$$

$$\therefore 1=A_1$$

$$A_1COS\sqrt{Q}+A_2\sin\sqrt{Q}=1 \tag{3.1.5}$$

$$At \quad \theta=0 \quad and \quad y=1$$

$$0=A_1\cos\sqrt{Q(1)}+A_2\sin\sqrt{Q(1)}$$

$$A_1\cos\sqrt{Q}+A_2\sin\sqrt{Q}=0$$

$$A_2\sin\sqrt{Q}=-A_1\cos\sqrt{Q}$$

$$A_2=-\frac{A_2\cos\sqrt{Q}}{\sin\sqrt{Q}} \tag{3.1.6}$$

$$\therefore\theta=A_1\cos\sqrt{Qy}+A_2\sin\sqrt{Qy}$$

From equation (3.1.1)

$$\frac{\partial^2u}{\partial y^2}-M^2U=-Gr\sin(\alpha)\theta$$

$$\frac{\partial^2u}{\partial y^2}-M^2U=-Gr\sin(\alpha)\left(A_1\cos\sqrt{Qy}+A_2\sin\sqrt{Qy}\right)$$

$$\frac{\partial^2u}{\partial y^2}-M^2U=-Gr\sin(\alpha)A_1\cos\sqrt{Qy}-Gr\sin(\alpha)A_2\sin\sqrt{Qy} \tag{3.1.7}$$

For homogeneous

$let \quad Up = A_5 \cos\sqrt{Qy} + A_6 \sin\sqrt{Qy}$

$$\frac{\partial^2 u}{\partial y^2} - M^2 U = 0$$

$$\frac{\partial^2 u}{\partial y^2} - M^2 U = 0$$

$$r^2 - m^2 u = 0$$

$$r^2 = m^2$$

$$r^2 = \pm m$$

$$Uh = A_3 \cosh M + A_4 \sinh M$$

$$\left. \begin{array}{l} Up = A_5 \cos\sqrt{Qy} + A_6 \sin\sqrt{Qy} \\ U'p = A_5 \cos\sqrt{Q} + A_6 \sin\sqrt{Q} \end{array} \right\} \quad (3.1.8)$$

$$U''p = 0$$

Substitution (3.1.8) from (3.1.7)

$$0 - M^2\left(A_5 \cos\sqrt{Qy} + A_6 \sin\sqrt{Qy}\right) = -Gr\sin(\alpha) A_1 \sin\sqrt{Qy} - Gr\sin(\alpha) A_2 \sin\sqrt{Qy}$$

$$-M^2 A_5 \cos\sqrt{Qy} = -Gr\sin(\alpha) A_1 \cos\sqrt{Qy}$$

$$M^2 A_5 = -Gr\sin(\alpha) A_1 \cos\sqrt{Qy}$$

$$A_5 = \frac{Gr\sin(\alpha) A_1}{M^2}$$

$$-M^2 A_6 \sin\sqrt{Qy} = -Gr\sin(\alpha) A_2 \sin\sqrt{Qy}$$

$$M^2 A_6 + Gr\sin(\alpha) A_2$$

$$A_6 = \frac{Gr\sin(\alpha) A_2}{m^2}$$

$$U = Uh + Up$$

$$A_3 \cosh M + A_4 \sinh M + A_5 \cos\sqrt{Qy} + A_6 \sin\sqrt{Qy}$$

$$U = A_3 \cosh M + A_4 \sinh M + A_5 \cos\sqrt{Qy} + A_6 \sin\sqrt{Qy}$$

Apply boundary condition

$At \quad U = 0 \quad y = 0$

$$0 = A_3 \cosh\sqrt{M(0)} + A_4 \sinh\sqrt{M(0)} + A_5 \cos\sqrt{Q(0)} + A_6 \sin\sqrt{Q(0)}$$

$$0 = A_3 + 0 + A_5 + 0$$

$$0 = A_3 + A_5$$

$$A_3 = -A_5$$

$$where \quad A_5 = \frac{Gr \sin(\alpha) A_2}{M^2}$$

$$At \qquad u = 1 \quad y = 1$$

$$1 = A_3 \cosh\sqrt{M(1)} + A_4 \sinh\sqrt{M(1)} + A_5 \cos\sqrt{Q(1)} + A_6 \sin\sqrt{Q(1)}$$

$$A_4 \sinh M = 1 - A_3 \cosh M - A_5 \cosh\sqrt{Q} - A_6 \sin\sqrt{Q}$$

$$But \qquad A_3 = -A_5$$

$$\therefore A_4 \sinh\sqrt{M} = 1 + A_3 A_5 \cosh\sqrt{M} - A_5 \cos\sqrt{Q} - A_6 \sin\sqrt{Q}$$

$$A_4 = \frac{1 + A_3 A_5 \cosh M - A_5 \cos\sqrt{Q} - A_6 \sin\sqrt{Q}}{\sinh\sqrt{Q}}$$

$$U = A_3 \cosh(My) + A_4 \sinh(My) + A_5 \cos\left(\sqrt{Qy} +\right) A_6 \sin\left(\sqrt{Qy}\right)$$

4. RESULTS AND DISCUSSION

In this section, we discuss the details of the investigations and the results obtained.

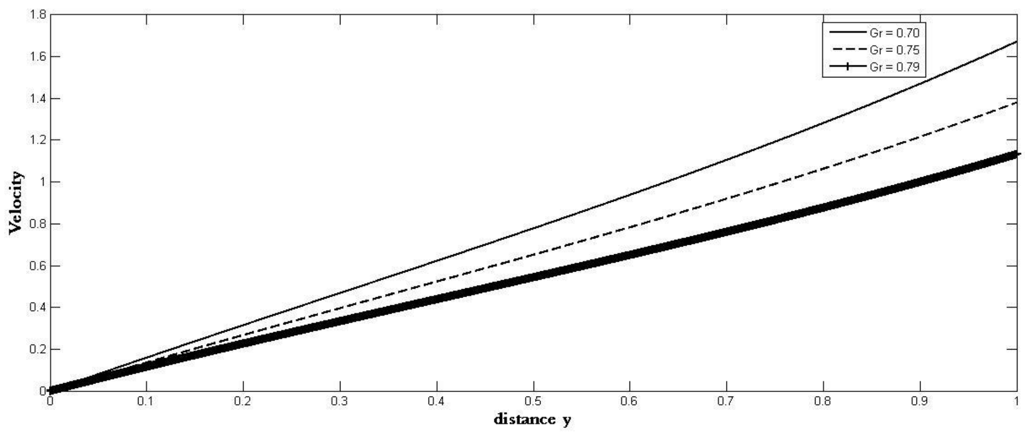

Fig 4.1 Effect of the crash of number (Gr) on the velocity profile

The above figure shows that the velocity profile decreases with an increase in the crash of number (Gr).

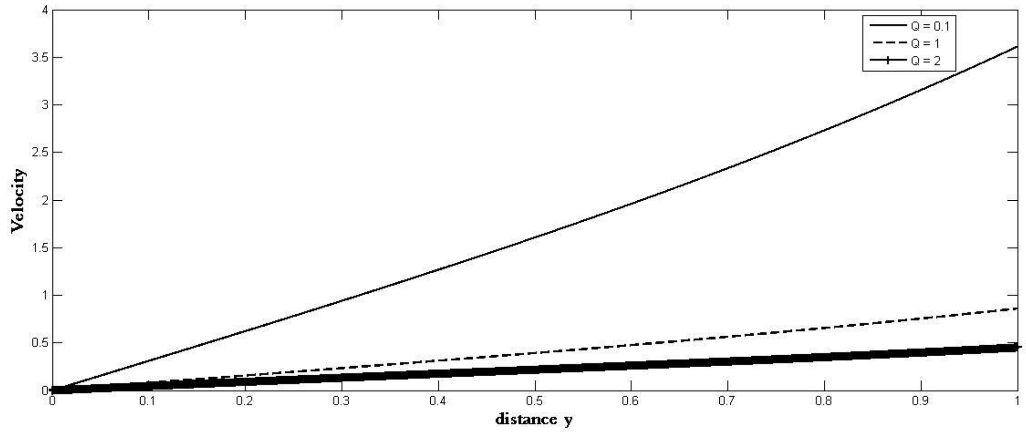

Fig 4.2 The Effect of Heat source (Q) on velocity

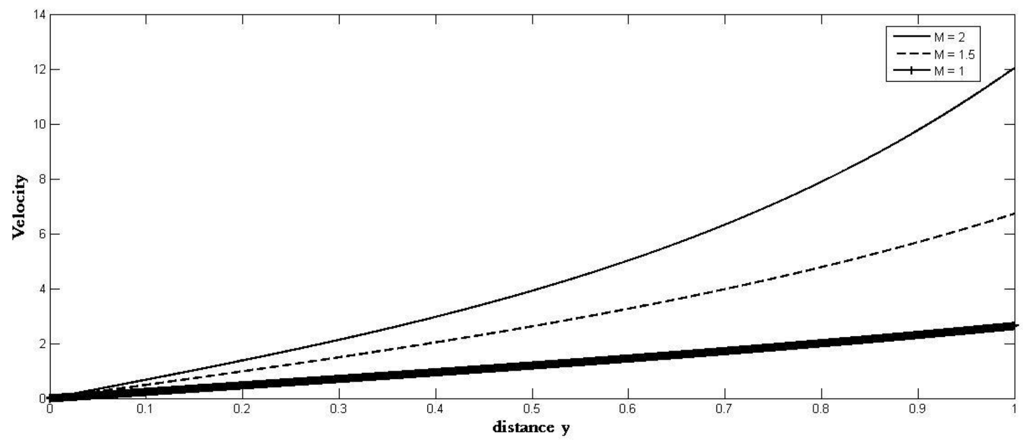

Fig 4.3 Effect of Magnetic field (M)

In Fig. 4.3, It seems that the velocity of the magnetic field parameter (M) decreases with an increase in Heat source (M)

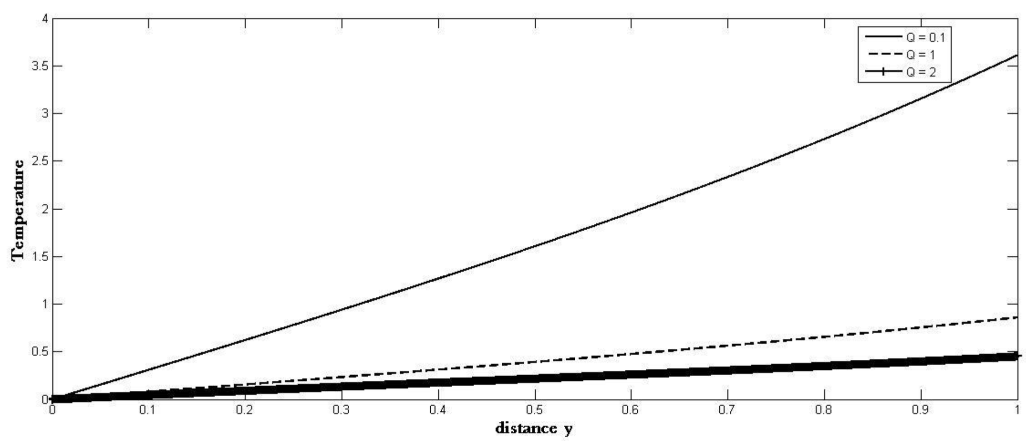

Fig 4.4 The Effect of Heat source (Q) on temperature.

In Figure 4.4, the temperature seems to decrease with an increase in Heat source (Q).

5. CONCLUSION

This paper analyzes the problem of MHD counterflow in an inclined vertical channel with a heat source. The equations governing the flow are solved analytically by using the perturbation method. The result of the computations is displayed graphically. This research result comprehensively overviews the fundamentals of partial differential equations (ODEs). This decreases with an increase in Grashof number (Gr) and decreases with an increase in heat source (Q) and magnetic field (M) on velocity. Also, the temperature decreases with an increase in heat source.

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