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SPATIAL-TEMPORAL MODEL FOR CAPTURING THE AUTOCORRELATION AND INTERACTION EFFECT IN MEASLES DATA

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ABSTRACT
Spatial data are data observed from different neighboring but non-overlapping areal units. In Nigeria, measles continues to be a leading cause of morbidity and mortality in children. Numerous factors, including spatial (S.P.) and temporal trends impacting the disease's intensity and dissemination, affect the transmission dynamics of measles. Although numerous methods exist for spatially time-to-event data, most S.P. models exclude the interaction effect and spatial autocorrelation. Therefore, this study offers the most effective model for overcoming these constraints and allows researchers to flexibly represent spatial clustering in their disease data. However, this research aims to use a–temporal mode to capture the autocorrelation and interaction effects in measles data. To provide an appropriate model for capturing the autocorrelation and interaction effect in the data across the areal units and time and to determine whether the model investigated above is consistent with capturing the measles data, This work uses simulations and poison distribution to identify the substantial spatiotemporal model and interaction/ autocorrelation influence on area unit data. The findings show that the interaction between the small area unit and period decreases as time increases, while the autocorrelation increases for the five area units. The spatial-temporal model is better fitted to 5 areal units when the period is 50 (5x50); however, in 10 and 15 area units, the interaction between the areal unit and period decreases as time increases, and the autocorrelation also decreases. The spatio-temporal model was better fitted to 10 areal units when the period was 20 (10 × 20) and 15 areal units when the period was 100 (15 × 10).

Keywords: Spatial-temporal data, Simulation, Areal units, Modeling, Poison prior

1. INTRODUCTION
Measles is a severe threat in many countries, including Nigeria. Numerous factors, including regional and temporal trends, impact the disease's intensity and dissemination. A comprehensive understanding of the spatiotemporal dynamics of measles is lacking, which hinders the development of effective control and prevention strategies. Measles is a severe public health threat in many countries, including Nigeria. This is because there isn't a robust modeling framework that can faithfully represent the temporal and spatial dynamics of the disease.

Meanwhile, current measles research frequently concentrates on discrete elements like disease burden or vaccine coverage but does not include these elements in a cohesive spatiotemporal model. This information gap hinders our ability to identify high-risk areas, understand transmission patterns, and develop targeted interventions to stop the spread of measles effectively. There are

additional obstacles in adequately estimating the spatial-temporal trends of measles due to a location's geographic characteristics, such as its diversified population distribution, varying accessibility to healthcare, and traditional cultural customs. These variables could be involved in localized epidemics and irregular disease transmission patterns in various places and areas.

Scan statistics, using spatio-temporal analyses, have been widely applied to explore disease patterns and detect high-risk disease clusters, particularly for diseases with limited resources for prevention (Kulldorff *et al.* 2015). Some studies have explored irregularly shaped disease clusters, which might more realistically reflect actual, albeit complicated, disease patterns (Costa and Kulldorff, 2014). Moreover, most studies have utilized relative risk (R.R.) derived from scan statistics to identify high-risk regions (Carrel *et al.* 2009, Jones *et al.* 2012, Ling *et al.*2014 and Sindato *et al.* 2014). Rare studies have measured the population-attributable risk percentage to evaluate the regional disease burden (Zhu *et al.*, 2013). Moreover, there is a lack of studies on the spatio-temporal patterns of measles, mainly in Guangxi, a region with large-scale measles outbreaks.

Lawson (2018), Rushworth *et al.* (2014), Napier *et al*. (2018) and Beraha *et al*. (2021) in their study, examined the issue of spatially dependent areal data and suggested using a finite mixture of Gaussian distributions to characterize the data gathered for every areal unit. A joint distribution for a set of vectors on the simplex, a logistic transformation of the Gaussian multivariate CAR models, is used to introduce spatial dependency.

A Bayesian model for spatiotemporal data based on a generalized linear mixed-effects model was suggested by Nicoletta *et al.* (2022) to quantify the percentage of isolation among various clusters in a given geographic area (Lee *et al*. 2018, 2021 and 2022) but did not consider interaction and autocorrelation effect.

A few of the many reasons to model these include determining the degree of segregation between two or more different groups in a town (Lee *et al*. 2018), identifying clusters of nearby area units that show an elevated risk of disease compared with neighboring areas (Anderson, Lee, and Dean 2014), and estimating the effect of a risk factor on a response. Frequently, sets of auto-correlated random effects are employed to replicate the spatio-temporal autocorrelation in these data through a Bayesian inferential technique. These random effects are typically allocated with spatio-temporal extensions of Conditional Autoregressive (CAR) priors to capture this autocorrelation. This study offers Bayesian hierarchical models using the poison distribution priors to predict the spatio-temporal data on associated simulated data and measles data.

Although there are numerous methods for dealing with spatial (S.P.) time-to-event data (e.g., Banerjee and Carlin, 2004. Choi and Lawson, 2011; Zhou *et al*., 2020; Lee and Pettersson, 2021), Conversely This Study, therefore, extends the work of Lee and Pettersson, 2021 where existing S.P. models either ignore spatial autocorrelation and interaction effect altogether. Thus, we shall present the best model that has the power to overcome these limitations as well as the flexibility for researchers to model spatial clustering in their disease data. This study, therefore, provides Bayesian hierarchical models for estimating the spatio-temporal data from related simulated data.

This study is significant because of its wide applications. The model consideration will help investigate, measure, analyze, and model spatial data features, such as places, characteristics, and their connections, that disclose the geometric or geographic characteristics of measles infections and other relevant data. The findings will assist in supplying the data that pinpoints the locations of features and boundaries where cases of measles are much more prevalent over time. Furthermore, this study will allow experts in the fields of science, sociology, psychology, epidemiology, biology, business, and marketing (among others) to understand better human behaviors, environmental factors, and relationships between people and a location.

1.1. Research Question

There is an urgent need for a comprehensive study that addresses the following research questions:

i. Is the Bayesian Hierarchical spatio-temporal model appropriate for capturing the autocorrelation and interaction effect in the data for poison prior across the areal units and periods?
ii. Is the model investigated above consistent with capturing the measles data simulated from different regions at consecutive periods and distribution

1.2 Aim and Objectives

The objectives of the study are

i. To provide an appropriate for capturing the autocorrelation and interaction effect in the data for poison prior across the areal units and periods
ii. To determine whether the model investigated above is consistent in capturing the measles data simulated from different regions at consecutive periods and distribution

2. RESEARCH METHODOLOGY

The model's effectiveness described in the preceding section is evaluated using a simulation procedure, focusing on the models' capacity to accurately estimate the varying degrees of temporal trend and geographical variability. Simulated measles incidence data are generated from binomial, poison and negative binomial distributions for the K = 5, 10 and 15 areal units and t =20, 50, 100 and 200 time periods. The areal units and periods are varied to assess the models' performance. The logit probability surface is generated from multivariate normal distributions.

The K × K neighborhood matrix $W = (w_{kj})$ where W is K x K matrix,

$$W = (w_{kj}) = \begin{pmatrix} w_{11} & \cdots & w_{1k} \\ \vdots & \ddots & \vdots \\ w_{k1} & \cdots & w_{kk} \end{pmatrix}$$

It is used in all models in this study to induce spatial autocorrelation into the response data Y through the latent component M. W typically consists of binary elements, where the value of $w_{kj} = 1$ if areal units (S_k, S_j) share a common border (i.e., are spatially close) and is zero otherwise. Additionally, when the value of $w_{kj} = 0$. This indicates that spatially adjacent areal units (S_k, S_j), (M_{kt}, M_{jt}) are spatially auto-correlated. In contrast, values for non-neighboring areal units are conditionally independent given the remaining $\{M_{it}\}$ values. This binary specification of W based on sharing a

common border is the most commonly used for areal data, but CARBayesST only requires that W be symmetric and contain non-negative elements. Similarly, the model S.T.CARanova () employs a binary N × N temporal neighborhood matrix $D = (d_{tj})$, where$d_{tj} = 1$, if |j − t| = 1 and$d_{tj} = 0$, otherwise.

2.1 Research Design

This study will partition the region into K = 1...k (Where k = 5, 10 and 15 areal units). The areal units studied constitute different communities, and data on generated measles susceptibility was obtained over time periods T (where T= 20, 50, 100, and 200). Bayesian hierarchical logistic regression models with poison prior were used to fit and evaluate its performance on the simulated data to get the best among them.

2.2 Population, sample and sample technique of the study

The population of this study will comprise local government areas in Taraba state of Nigeria, and three local government areas will be considered from each of the three senatorial districts in Taraba state to make a total number of nine samples. Sample data on measles across the local governments were considered using random sampling. The data shall be collected across the region for 30 years or 30 months.

2.3Method of data collection

The data collection method will be simulations based on different distributions. Secondary data on measles cases will also be obtained from the general hospital across the selected local government area. The research will cover the following Local government areas in Taraba state: Bali, Zing, Wukari, Donga, Gassol, Jalingo, Gashaka, Takum and Lau.

A simulation study was carried out to evaluate the effect of poison, binomial, and negative binomial distributions described in the next sections, particularly the accuracy in estimating the varying degrees of temporal trend and spatial variability. Simulated measles incidence data are generated from poison distribution for the k = 5, 10 and 15 areal units and T= 20, 50, 100 and 200 times period. The population sizes are varied to assess the models' performance in case simulation.

2.4 Technique for Simulation and Data Analysis

The technique for the data analysis is Bayesian hierarchical logistic regression models with poison prior, which was used to model the simulated data and determine when the best is fitted across areal units and time periods. Our Bayesian hierarchical model has the following prior distribution.

$$Y_{kt} \sim pois(N_t, \theta_{kt})$$

$$\ln\left(\frac{\theta_{kt}}{1-\theta_{kt}}\right) = Y'_{kt}\beta + \delta_t + e_t \qquad (1)$$

The measles susceptibility logit probabilities are represented as a linear mixture of two random effects, a px1 vector Y_{kt} of covariates and their corresponding regression parametersβ, δ_t and e_t. Upon the regression parameters β, a multivariate normal prior distribution is applied,

$$\beta \sim N(0, I)$$

Where I_{pxp} is the identity matrix, after the covariate effects are considered, the random effects δt and et are added to the data to represent the temporal and spatial trends and autocorrelation. The temporal random effects come first. The random walk of order one prior distribution for e_t=e_1 e_2,…, e_N is provided as follows.

$$e_t \sim N(e_{t-1}, \sigma^2), t = 1,2, \dots, N$$

The $e_t$will be varied for different prior distributions as follows

$$e_t \sim Unif(e_{t-1}, \sigma^2), t = 1,2, \dots, N$$
$$e_t \sim exp(e_{t-1}, \sigma^2), t = 1,2, \dots, N$$
$$\sigma^2 \sim inversegamma(0.1,0.1)$$

The effect at the prior time period, e_{t-1}, is the mean of the temporal random effec$_t$ at time period t. Thus, the temporal random effects capture the overall temporal trend in the region's vulnerability to measles.

2.5 Spatio-Temporal Interaction Effect for areal unit data

The most common modeling approaches utilize a main effect, interaction effect or an autoregressive structure (Rushworth *et al.*, 2014). Different levels of this effect will be examined in the model. Data simulation techniques are used to estimate the number of susceptible measles cases (y_{it}) per year and the number of children visiting the hospital (n_{it}). The interaction effect (γ_{kt}) is incorporated in (1) to five (2) as follows

$$\ln\left(\frac{\theta_{kt}}{1-\theta_{kt}}\right) = X'_{kt}\beta + \delta_t + \gamma_{kt} + e_t \qquad (1)$$

$$\gamma_{kt/\mu_t \sim N(0,\mu_t^2)}$$

Where θ_{kt} Is the estimated likelihood of having measles at a particular region and time? The remaining terms in the linear predictor are $\delta = (\delta_1, \delta_2, .. \delta_N)$ and $e = (e_1, e_2, \dots, e_N)$. What are the overall spatial and temporal trends in the estimated probability?θ_{kt}.

$\gamma_{kt/\mu_t \sim N(0,\mu_t^2)}$ As an interaction effect, its mean values also varied in the simulations to investigate the strength of the spatio-temporal interaction effect on the data.

2.6 Spatio-Temporal Models for M_{kt}

Using a K × K neighborhood matrix W = (w_{kj}), all models in this study produce spatial autocorrelation in the response data Y through the latent component M. W typically contains binary elements, where w_{kj} = 0 otherwise and one if areal units (S_k, S_j) are spatially close to one another or share a shared border. Furthermore, w_{kk} = 0. This indicates that while values for non-neighboring areal units are conditionally independent given the remaining {M_{it}} values, those for geographically nearby areal units (S_k, S_j) have a spatial auto-correlated relationship (M_{kt}, M_{jt}). The most popular binary specification of W for areal data is this one, which is based on having a shared boundary; however, CARBayesST requires that W be symmetric and have non-negative elements.

The spatial and temporal main effects and space-time interaction model Knorr-Held (2000) put forth are similar to the S.T.CARanova model. The model is considered to fit our simulated data of

different distribution categories, parameter values/ levels, levels of an interaction effect, autocorrelation levels and sample sizes to estimate the susceptibility of measles across the regions. The details of the model's description are given below.

2.7 S.T.CARanova

The model is a modification of that proposed by Knorr-Held (2000) and is given by

$$M_{kt} = \emptyset_k + \delta_t + \gamma_{kt} \qquad (2)$$

$$\emptyset_k \sim N\left(\frac{\rho_S \sum_{j=1}^{k} w_{kj}\emptyset_j}{\rho_S \sum_{j=1}^{k} w_{kj}\emptyset_j + 1 - \rho_S}, \frac{\tau_S^2}{\rho_S \sum_{j=1}^{k} w_{kj}\emptyset_j + 1 - \rho_S}\right)$$

$$\delta_k \sim N\left(\frac{\rho_T \sum_{j=1}^{k} w_{kj} d_{tj}\delta_j}{\rho_T \sum_{j=1}^{k} d_{tj} + 1 - \rho_T}, \frac{\tau_T^2}{\rho_T \sum_{j=1}^{k} d_{tj} + 1 - \rho_T}\right)$$

$$\tau_S^2, \tau_T^2, \tau_I^2 \sim Inverse - Gamma(a, b),$$

$$\rho_S, \rho_T \sim Uniform(0, 1),$$

$$\gamma_{kt} \sim N(\mu_\alpha, \tau_I^2).$$

In this case, the CAR prior suggested by Leroux et al. (2000) models both the spatio-temporal autocorrelation and a standard set of spatial random effects, $\emptyset = (\emptyset_1,...,\emptyset_k)$, and a standard set of temporal random effects, $\delta = (\delta_1,...,\delta_k)$. Furthermore, the model can optionally include a collection of separate space-time interactions $\gamma = (\gamma_{11...}\ \gamma_{kt})$, which can be indicated in the function call by passing in the argument interaction=TRUE (the default). Every random effect set has a mean center. For the remaining parameters, fixed uniform (ρ_S, ρ_T) and conjugate $(\tau_S^2, \tau_T^2, \tau_I^2 I)$ priors are supplied; default values for the latter are (a = 1, b = 0.01). Alternatively, instead of estimating the dependence parameters (ρ_S, ρ_T) in the model, they can be fixed at values in the unit interval [0, 1]

4. ANALYSIS AND RESULTS

4.1. Model Fitting and Summary of the Numerical Results on 5-Areal unit when there is Interaction and Autocorrelation effect

The simulation for 5-arealUnit at different periods is carried out to fit the spatial-temporal model and to illustrate the appropriateness of the model fitting across the arealUnit and period to establish the objective. We assume here that the data comes from Poisson likelihood. Before making an inference from the model, we have to ensure the Markov chains appear to have converged, and a single chain diagnostic is the Z-score proposed by Geweke (2022) and given in the model summary in the table.1 (Geweke. diag), where convergence is suggested if the Z-score lies within (−1.96, 1.96). Table 1 includes parameter estimates (Mean values), 95% credible intervals (2.5% and 97.5%) of the parameter and convergence diagnostics (n effective and Geweke. diag) for specific parameters, as well as overall model fit measures such as the DIC. We assume here that the interaction terms γkt are absent and that the data come from a Poisson likelihood. Where τ_s *and* τ_T The estimated probability of having measles is at a particular region and time. ρ_s and ρ_T These are the interactions between an areal unit and time period and the autocorrelation effect.

Table 1: Spatio-temporal models for five areal unit data over 20, 50, 100 and 200 time periods with interaction and autocorrelation effect

K x T	T	Mean	2.5%	97.5%	n.effective	Geweke.diag	DIC	p.d	LMPL
5X20	Intercept	4.0377	4.0316	4.0437	4787.8	0.6	13911.7	96.237	-6958.89
	τ_s	0.0094	0.0065	0.0130	5200.9	1.7			
	τ_T	0.0132	0.0070	0.0243	5000.0	0.3			
	ρ_s	0.9281	0.7949	0.9940	4544.9	-1.1			
	ρ_T	0.7169	0.3050	0.9654	5000.0	-0.7			
5X50	Intercept	3.9600	3.9522	3.9679	5000.0	0.1	**8674.58**	**68.489**	**-4341.8**
	τ_s	0.0092	0.0055	0.0206	4860.3	-0.3			
	τ_T	0.0116	0.0102	0.0244	4607.9	-1.0			
	ρ_s	0.6957	0.0689	0.9247	4553.3	1.6			
	ρ_T	0.8176	0.5829	0.9624	4733.8	0.5			
5X100	Intercept	3.8549	3.8491	3.8608	5000.0	0.7	16833.6	108.01	-8418.3
	τ_s	0.0090	0.0049	0.0160	5000.0	1.6			
	τ_T	0.0088	0.0061	0.0124	4600.3	-1.4			
	ρ_s	0.6393	0.2085	0.9570	5000.0	1.1			
	ρ_T	0.9507	0.8822	0.9905	4457.5	0.4			
5X200	Intercept	3.9603	3.9560	3.9644	4138.0	-1.6	34135.9	198.57	-17069.3
	τ_s	0.0078	0.0042	0.0137	5000.0	0.4			
	τ_T	0.0068	0.0077	0.0123	4597.5	1.0			
	ρ_s	0.6127	0.1758	0.9524	4817.2	-1.0			
	ρ_T	0.9864	0.9688	0.9971	5000.0	-0.8			

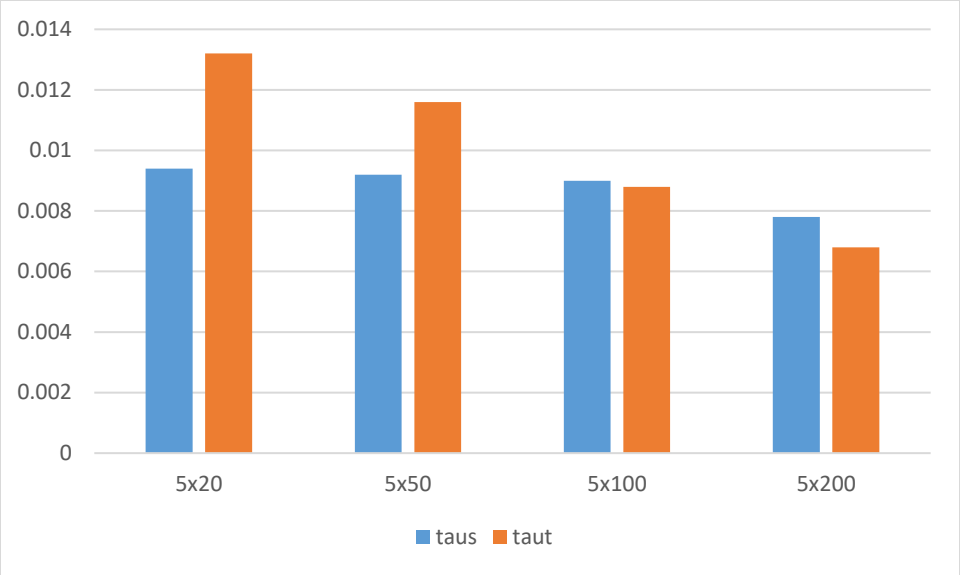

Fig.1a: Estimated parameter values of measles from 5 areal units over some time

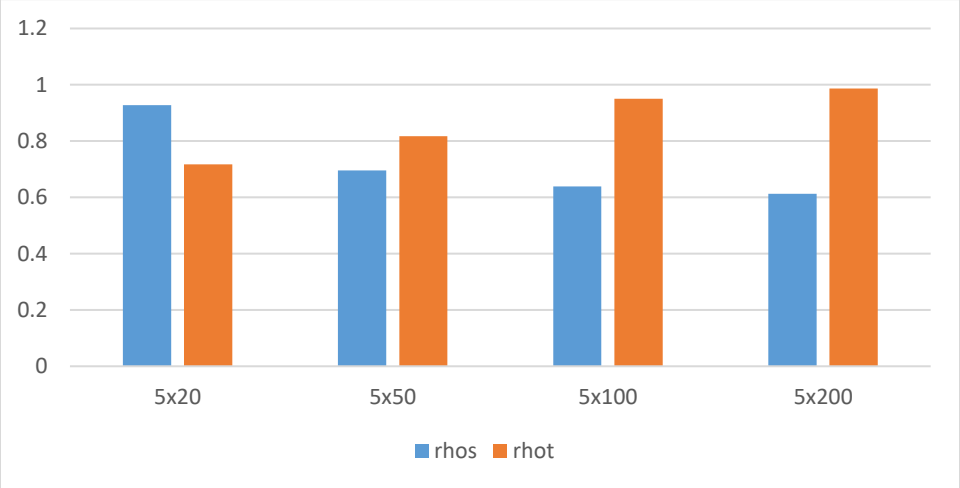

Fig.1b: Estimated interaction and autocorrelation of 5 areal units and periods

The table above shows the output describing the model fitted on five areal unit data at t = 1, 2, 3… 20; 50; 100 and 200 time periods, and a summary of the numerical results. The estimated Values of the measles susceptibility at a particular region and time (τ_s and τ_T) is presented in Figure 1a, while the correlation between an areal unit and period and autocorrelation effect, respectively (ρ_s andρ_T) is displayed in Figure 1b. From Figure 1a, both values of τ_s and τ_T These decrease as the period increases by five units. Indeed, the estimated measles susceptibility decreases over time. Figure 1b shows that the ρ_s decreases while ρ_T Increases over the period. This shows that the interaction between small arealUnit and period decreases as time increases while the autocorrelation increases. The spatial-temporal model is more fitted to 5 areal units when the period is 50 (5x50) due to its minimum DIC and P.D. and maximum value of LMPL.

4.2 Model Fitting and Summary of the Numerical Results on 10-Areal unit when there is Interaction and Autocorrelation effect

The simulation for 10-arealUnit at different periods is carried out to fit the spatial-temporal model and to illustrate the appropriateness of the model fitting across the arealUnit and period to establish the objective. We assume here that the data comes from Poisson likelihood. Before making an inference from the model, we have to ensure the Markov chains appear to have converged. A single chain diagnostic is the Z-score proposed by Geweke(2022) and given in the model summary in Table 2 (Geweke. diag), where convergence is suggested if the Z-score lies within (−1.96, 1.96). Table 2b includes parameter estimates (Mean values), 95% credible intervals (2.5% and 97.5%) of the parameter and convergence diagnostics (n effective and Geweke. diag) for specific parameters, as well as overall model fit measures such as the DIC. We assume here that the interaction terms γkt are absent and that the data come from a Poisson likelihood. Where τ_s *and* τ_T are estimated probability of having measles at a particular region and time, respectively ρ_s and ρ_T Are interactions between an areal unit and period and autocorrelation effect, respectively?

Table 2: Spatio-temporal models for ten areal unit data over 20, 50, 100 and 200 time periods with interaction and autocorrelation effect from poison prior

KXT	T	Mean	2.5%	97.5%	n.effective	Geweke.diag	DIC	p.d	LMPL
10X20	Intercept	4.221	4.2151	4.2268	5000.0	1.0	**14233.6**	**103.68**	**-7119.0**
	τ_s	0.012	0.0087	0.0170	5000.0	0.6			
	τ_T	0.014	0.0073	0.0287	4795.4	0.2			
	ρ_s	0.868	0.6593	0.9863	4409.0	-0.8			
	ρ_T	0.955	0.8480	0.9973	4036.7	0.1			
10X50	Intercept	4.241	4.2384	4.2454	3692.5	-0.4	35667.6	142.00	-17836.9
	τ_s	0.017	0.0087	0.0168	5000.0	-0.8			
	τ_T	0.012	0.0078	0.0179	5000.0	-0.3			
	ρ_s	0.744	0.4627	0.9615	5000.0	-1.7			
	ρ_T	0.864	0.6786	0.9751	4726.2	-0.3			
10X100	Intercept	4.241	4.2384	4.2454	3692.5	-0.4	35667.6	142.00	-17836.9
	τ_s	0.018	0.0087	0.0168	5000.0	-0.8			
	τ_T	0.011	0.0078	0.0179	5000.0	-0.3			
	ρ_s	0.634	0.4627	0.9615	5000.0	-1.7			
	ρ_T	0.864	0.6786	0.9751	4726.2	-0.3			
10X200	Intercept	4.241	4.2384	4.2454	3692.5	-0.4	35667.6	142.00	-17836.9
	τ_s	0.018	0.0087	0.0168	5000.0	-0.8			
	τ_T	0.009	0.0078	0.0179	5000.0	-0.3			
	ρ_s	0.511	0.4627	0.9615	5000.0	-1.7			
	ρ_T	0.864	0.6786	0.9751	4726.2	-0.3			

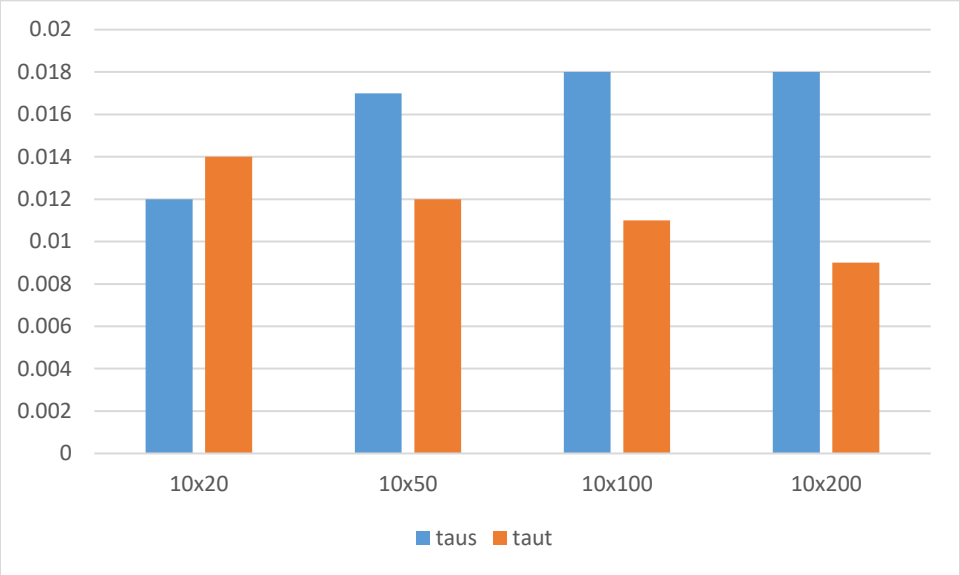

Fig.2a: Estimated parameter values of measles from 10 areal units over some time

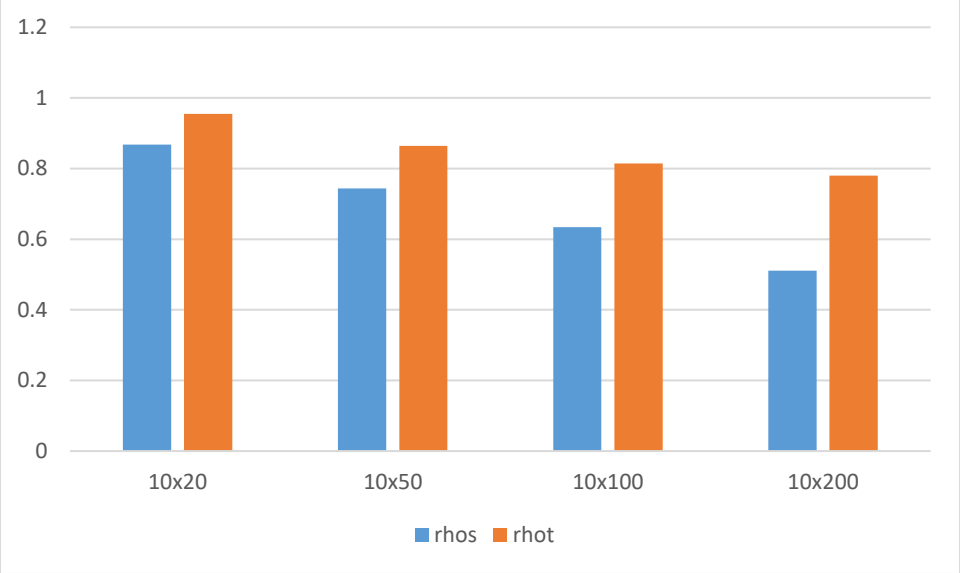

Fig.2b: Estimated interaction and autocorrelation of 10 areal units and periods

Table 2 shows the output that describes the model fitted on ten areal unit data at t = 1, 2, 3… 20; 50; 100 and 200 time periods, and a summary of the numerical results. The estimated values of the measles susceptibility at a particular region and time (τ_s *and* τ_T) is presented in Figure 2a, while interaction between an areal unit and period and autocorrelation effect, respectively ($\rho_s$andρ_T) is displayed in Figure 2b. From figure2a, values of τ_s *is increasing while* τ_T They decrease as the period increases unit. Units Indeed, the estimated measles susceptibility increases over time. Figure 2b shows that both values of ρ_s and ρ_T Decreases over period. This shows that the interaction between small arealUnit and period decreases as time increases, and the autocorrelation also

decreases. The spatial-temporal model is more fitted to 10 areal units when the time period is 20 (10x20) due to its minimum DIC and p.d and maximum value of LMPL.

4.3. Model Fitting and Summary of the Numerical Results on 15-Areal Unit data over 20, 50, 100 and 200 time periods with interaction and autocorrelation effect

The simulation for 15-arealUnit at different periods is carried out to fit the spatial-temporal model and to illustrate the appropriateness of the model fitting across the arealUnit and period to establish the objective. We assume here that the data comes from Poisson likelihood. Before making an inference from the model, we have to ensure the Markov chains appear to have converged. A single chain diagnostic is the Z-score proposed by Geweke(2022) and given in the model summary in Table 3 (Geweke. diag), where convergence is suggested if the Z-score lies within (−1.96, 1.96). Table 3 includes parameter estimates (Mean values), 95% credible intervals (2.5% and 97.5%) of the parameter and convergence diagnostics (n effective and Geweke. diag) for specific parameters, as well as overall model fit measures such as the DIC. We assume here that the interaction terms γkt are absent and that the data come from a Poisson likelihood. Where τ_s *and* τ_T Are estimated probability of having measles at a particular region and time respectively. ρ_s and ρ_T The interaction between an areal unit and period and the autocorrelation effect are as follows:

Table 3: Spatio-temporal models for 15 areal unit data over 20, 50, 100 and 200 time periods with interaction and autocorrelation effect from poison prior.

KXT	T	Mean	2.5%	97.5%	n.effective	Geweke.diag	DIC	p.d	LMPL
15X20	Intercept	4.178	4.174	4.1816	4790.8	-1.2	31701.2	189.29	-15851.7
	τ_s	0.007	0.0057	0.0093	4380.9	-1.7			
	τ_T	0.010	0.0055	0.0185	5000.0	1.7			
	ρ_s	0.839	0.6480	0.9744	4423.0	-0.6			
	ρ_T	0.570	0.1251	0.9233	4584.4	-0.8			
15X50	Intercept	4.074	4.1555	4.3683	4534.2	0.7	25849	123.76	-12894.1
	τ_s	0.008	0.0523	0.0825	5000.5	-1.6			
	τ_T	0.010	0.1567	0.0131	4054.8	0.9			
	ρ_s	0.822	0.6259	0.9107	3815.1	-0.5			
	ρ_T	0.557	0.0362	0.4249	4275.4	-0.7			
15X100	Intercept	3.953	3.9474	3.9596	4554.5	0.6	**5778.90**	**287.17**	**-2598.6**
	τ_s	0.009	0.0066	0.0130	5000.0	-1.3			
	τ_T	0.006	0.0034	0.0116	5000.0	1.8			
	ρ_s	0.816	0.6049	0.9014	4012.0	0.4			
	ρ_T	0.513	0.0768	0.9047	4000.0	-0.4			
	Intercept	4.035	4.0261	4.0444	4703.7	1.6	6921.44	148.61	-3457.63

15X200	τ_s	0.010	0.0047	0.0120	3956.2	0.1			
	τ_T	0.018	0.0066	0.0464	5000.0	0.1			
	ρ_s	0.590	0.2248	0.9168	3701.9	-1.4			
	ρ_T	0.499	0.0260	0.3939	3929.0	0.0			

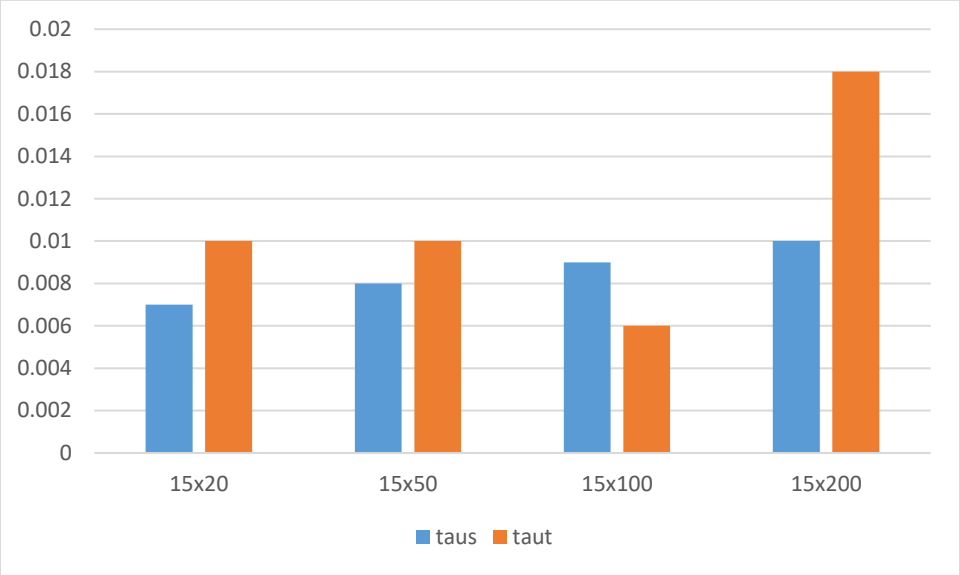

Fig.3a: Estimated parameter values of measles from 15 areal units over some time

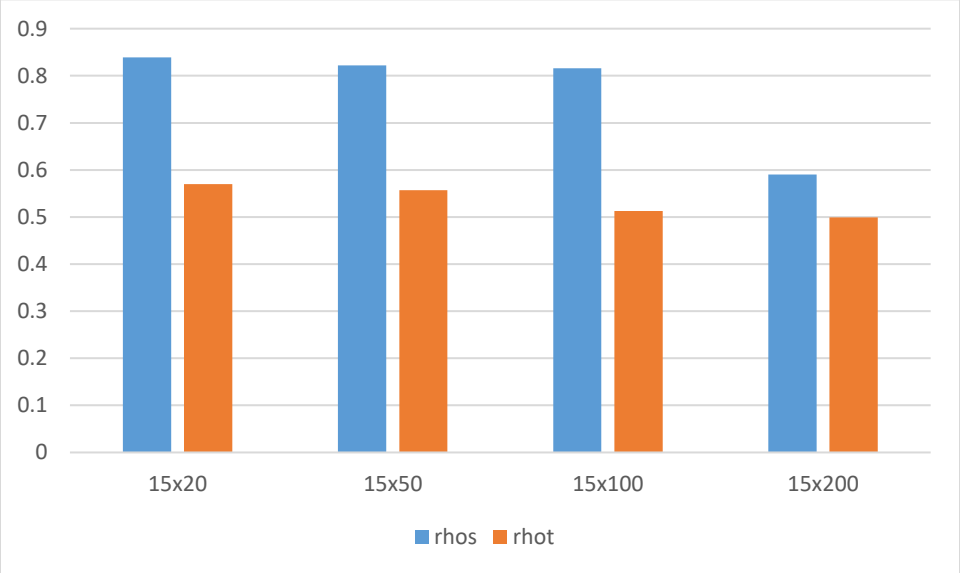

Fig.3b: Estimated interaction and autocorrelation of 15 areal units and periods

Table 3 shows the output that describes the model fitted on 15 areal unit data at t = 1, 2, 3… 20; 50; 100 and 200 time periods, and a summary of the numerical results. The estimated values of the

measles susceptibility at a particular region and time (τ_s and τ_T) is presented in Figure 3a, while interaction between an areal unit and period and autocorrelation effect, respectively ($\rho_s$andρ_T) is displayed in Figure 3b. From Figure 4.3a, both values of τ_s and τ_T Are increasing as the period increases by 15 areal units. Indeed, the estimated measles susceptibility increases over time. Figure 4.3b shows that both values of ρ_s and ρ_T Decreases over period. This shows that the interaction between small arealUnit and period decreases as time increases, and the autocorrelation also decreases. The spatial-temporal model is more fitted to 15 areal units when the period is 100 (15x100) due to its minimum DIC and p.d and maximum value of LMPL.

5. CONCLUSION

This study provided an appropriate model for capturing the autocorrelation and interaction effects in the spatiotemporal data for poison prior across the areal units and periods. It determined a suitable model for measles-generated data from non-overlapping regions at consecutive periods. The distributions through simulation with the following result achieved. The findings show that the interaction between small area units and periods decreases as time increases while the autocorrelation increases for 5 area units. The spatial-temporal model is more fitted to 5 areal units when the period is 50 (5x50) due to its minimum DIC and P.D. and maximum value of LMPL. Also, in 10 area units, the interaction between small area units and periods decreases as time increases, and autocorrelation decreases. The spatial-temporal model is more fitted to 10 areal units when the period is 20 (10x20). Finally, under 15 areal units, the interaction between small arealUnit and period decreases as time increases, and the autocorrelation also decreases. The spatial-temporal model is more fitted to 15 areal units when the period is 100 (15x100) due to its minimum DIC and p.d and maximum value of LMPL.

6. LIMITATIONS OF THE STUDY

This study is limited to areal units partitioned into K = 1... k (Where k = 5, 10, 15 areal units). The region under study constitutes different communities with data obtained over periods t (where t = 1, 2, 3…). To get the best among them, Bayesian hierarchical logistic regression models with only poison prior were used to model the simulated measles data. The simulations based on the earlier distribution were carried out, and the model was assessed.

7. IMPORTANCE OF FINDINGS

This study is significant because of its wide applications. To identify the geometric or geographic characteristics of measles diseases and other relevant data, the model under consideration will assist in investigating, assessing, evaluating, and modeling spatial data features, such as locations, qualities, and their interactions. The outcomes will help provide information that explains the characteristics and boundaries of the areas where measles cases are much more prevalent over time. Furthermore, this study will allow experts in the fields of science, sociology, psychology, epidemiology, biology, business, and marketing (among others) to understand better human behaviors, environmental factors, and relationships between people and a location.

8. FURTHER RESEARCH

Further research can use spatiotemporal model for fitting and forecasting in a real life data of measles from different areal units and regions at different time periods.

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