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SOME RESULTS ON NEW INTEGER-VALUED STATISTICS ON THE Γ_1-NON DERANGED PERMUTATIONS

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ABSTRACT
In this paper, we investigate the statistics of record, anti-record, exclusive record, and record-antirecord on the scheme called Γ_1- non deranged permutations, the permutation which fixes the first element in the permutations. This was done first through some computation on this scheme using prime numbers $p \geq 5$.the Statistics $_p$record, anti-record and record-antirecord are equal to one where $i = 1$. We also devise a method for calculating the sum of the four statistics on any $\omega_1 \in G_p^{\Gamma_1}$, other theoretic properties were also observed.

Keywords: Γ_1-non deranged permutations, Record, Antirecord, Exclusive record

1. INTRODUCTION

Equip distribution problems of set-valued statistics on permutations have attracted much attention in recent literature see [Baril and Vajnovszki(2017), Foata and Guo-Niu (2009), Kim and Lin(2018), Poznanović (2014)]. Aunu permutation pattern first arose, out of attempts to provide some combinatorial interpretations of some succession scheme and today series of results ranging from its avoiding class to its properties which have also been studied. Garba and Ibrahim (2010) developed a strategy for the prime numbers $p \geq 5$ and $\Omega \subseteq N$ employing the catalan numbers as well. This scheme creates a cycle of permutation patterns that is utilized to decide the arrangements. Researchers have over time looked also at permutation group with certain properties; one that comes to mind is the permutation patterns that have any of the element fixed or the one that has no fixed element, here the idea of deranged and non-deranged permutation surface. It is in line with this understanding that Ibrahim, Ejima and Aremu (2016) modified the scheme of Garba and Ibrahim (2010) to two line notation and the scheme generated a set of permutations with a fix at 1 (which generated the natural identity). This obtained set of permutations form permutation group called the Γ_1- non deranged permutation group and is denoted as $G_p^{\Gamma_1}$. Ibrahim, Ibrahim, Garba and Aremu (2017) outline the theoretical characteristics of the Ascent set in regard to the Γ_1-non deranged permutation and demonstrate that the union of the Ascent set equals the identity. They also note that the difference between $Asc(\omega_i)$ and $Asc(\omega_{p-1})$ is one. Aremu, Buoro, Garba and Ibrahim (2018) utilized the direct and skew sum operation on the components of the Γ_1-non deranged permutation group and showed the relationships and schemes on the structures and fixed point of the permutations generated from these operations. Additionally, if π is the direct sum of these Γ_1-non deranged permutations, then the collection of permutations in the form of π is an abelian group, designated as $G_p^{\Gamma_{m\oplus}}$. According to Aremu, Garba, Ibrahim and Buaro (2019), the $\mathrm{Re}\,s(\omega_i)$ and $\mathrm{Re}\,s(\omega_{p-1})$ of Γ_1-non deranged permutations are equally distributed between the right and left embracing numbers, respectively. Additionally, it notices that the height of the weighted motzkin path of ω_i is the same height as the height of $\omega_{p-des(\omega_i)}$ motzkin path. Ibrahim and Garba (2019) studied ascent and descent blocks of Γ_1 - non deranged permutations. moreover, it defined the mapping $\Psi_{AI}: G_p^{\Gamma_m} \to \Omega_p$ which converts the permutation from the weighted Motzkin path in the Ω_p with respect to the ascent and descent blocks from the Γ_1- non deranged permutations group $G_p^{\Gamma_m}$. Inversion number and major index are not equally distributed in Γ_1-non deranged permutations, as demonstrated by Garba and Ibrahim (2019), who also established that the difference between the sums of the major index and the inversion numbers is equal to the sum of the descent numbers in these permutations. Ibrahim and Muhammad (2019) produced recursion formulas for the maximum and lowest block counts in Γ_1 - non deranged permutations. They also noted that $arc(\omega_i)$ and $asc(\omega_i)$ are equally distributed in Γ_1-non deranged permutations. Using the membership function that was built for the fuzzy set on $G_p^{'}$, Ibrahim, Garba, Alhassan and Hassan (2021) investigated some algebraic theoretic properties of the fuzzy set on $G_p^{'}$ and established the results for the algebraic operators of the fuzzy set on $G_p^{'}$, which are the algebraic sum, algebraic product, bounded sum, and bounded difference. They also built a relationship between the operators and the fuzzy set on $\mathcal{G}_p{}'$. Ibrahim, Ibrahim and Ibrahim (2022) proved that the right embracing sum $\mathrm{Re}\,s(\omega_i)$ is equal to the right embracing sum $\mathrm{Re}\,s(\omega_{p-i})$ where $1 \le i \le p-1$ and $\mathrm{Re}\,s(\omega_i) = \frac{(p-3)(p-1)}{8}$ where $i = \frac{p-1}{2}, \frac{p+1}{2}$. It also observed that the left embracing sum $Le\,s(\omega_i)$ is equal to the right embracing sum $\mathrm{Re}\,s(\omega_i)$ and $Le\,s(\omega_{\frac{p+1}{2}}) = \frac{p^2-1}{8}$ where $p \ge 5$. More recently Ibrahim and Aremu (2023) established that the chromatic number of any $\chi(G(\omega_{P-I}))$ in $G_p^{\Gamma_1}$ is equal to $\mathrm{p}-1$ and any $\chi(G(\omega_I))$ in $G_p^{\Gamma_1}$ is equal to one. Similarly, the chromatic index of any $\chi'(G(\omega_{P-I}))$ in $G_p^{\Gamma_1}$ is equal to $\mathrm{p}-2$ and any $\chi'(G(\omega_I))$ in $G_p^{\Gamma_1}$ is equal to zero.

In this paper, we show that the Statistics record, anti-record and record-antirecord are equal to one where $i = 1$. We also show a devise method for calculating the sum of the four statistics on any $\omega_1 \in G_p^{\Gamma_1}$, other theoretic properties were also observed.

2. PRELIMINARIES

In this section, we give some definitions and preliminaries which are useful in our work.

Definition 2.1

Let Γ be a non-empty set of prime cardinality $p \geq 5$ such that $\Gamma \subset N$ A bijection ω on Γ of the form

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & (1+i)_{mop} & (1+2i)_{mop} & . & . & . & (1+(p-1)i)_{mop} \end{pmatrix}$$

is called a Γ_1-non-deranged permutation. We denoted G_p to be the set of all Γ_1-non deranged permutations.

Definition 2.2

The pair G_p and the natural permutation composition forms a group which is denoted as $G_P^{\Gamma_1}$. This is a special permutation group which fixes the first element of Γ.

Definition 2.3

Let $\omega_i \in G_p^{\Gamma_1}$. Then the record of ω_i denoted by $rec(\omega_i)$ is defined as follows:

$$rec(\omega_i) = \left| \{ (j, \omega_i(j) : \omega_i(k) < \omega_i(j), \quad \forall k < j) \} \right|.$$

Definition 2.4

Let $\omega_i \in G_p^{\Gamma_1}$. Then the anti-record of ω_i denoted by $arec(\omega_i)$ is defined as follows:

$$arec(\omega_i) = \left| \{ (j, \omega_i(j) : \omega_i(k) > \omega_i(j), \quad \forall k > j) \} \right|.$$

Definition 2.5

Let $\omega_i \in G_p^{\Gamma_1}$. Then the exclusive-record of ω_i denoted by $erec(\omega_i)$ is defined as follows:

$$erec(\omega_i) = \left| \{ (j, \omega_i(j) : \omega_i(k) < \omega_i(j), \quad \forall k < j) \} - \{ (j, \omega_i(j) : \omega_i(k) > \omega_i(j), \quad \forall k > j) \} \right|.$$

Definition 2.6

Let $\omega_i \in G_p^{\Gamma_1}$. Then the record-antirecord of ω_i denoted by $rar(\omega_i)$ is defined as follows:

$$rar(\omega_i) = \left| \{ (j, \omega_i(j) : \omega_i(k) < \omega_i(j), \quad \forall k < j) \} \cap \{ (j, \omega_i(j) : \omega_i(k) > \omega_i(j), \quad \forall k > j) \} \right|.$$

3. RESULTS AND DISCUSSION

In this section, we discuss the details of the investigations and results obtain.

Proposition 3.1

Let $\omega_1 \in G_p^{\Gamma_1}$. Then the

$$rec(\omega_1) = p.$$

Proof:

For any $\omega_i \in G_p^{\Gamma_1}$

The $rec(\omega_i) = \left|\{(j, \omega_i(j) : \omega_i(k) < \omega_i(j), \quad \forall k < j)\}\right|$. Since, for any $G_p^{\Gamma_1}$, $\omega_1 = \begin{pmatrix} 1 & 2 & . & . & . & p \\ 1 & 2 & . & . & . & p \end{pmatrix}$.

Then for all $k < j$, $\omega_1(k) < \omega_1(j)$. Therefore,

$$rec(\omega_1) = p. \qquad \square$$

Proposition 3.2

Let $\omega_1 \in G_p^{\Gamma_1}$. Then the

$$arec(\omega_1) = p.$$

Proof:

For any $\omega_i \in G_p^{\Gamma_1}$

The $arec(\omega_i) = \left|\{(j, \omega_i(j) : \omega_i(k) > \omega_i(j), \quad \forall k > j)\}\right|$. Since, for any $G_p^{\Gamma_1}$, $\omega_1 = \begin{pmatrix} 1 & 2 & . & . & . & p \\ 1 & 2 & . & . & . & p \end{pmatrix}$.

Then for all $k > j$, $\omega_1(k) > \omega_1(j)$. Therefore,

$$arec(\omega_1) = p. \qquad \square$$

Proposition 3.3

Let $\omega_1 \in G_p^{\Gamma_1}$. Then the

$$erec(\omega_1) = 0.$$

Proof:

By the Definition of exclusive record. Exclusive records are records that are not anti-records. By Proposition 3.1 and 3.2, all the elements in ω_1 are both records and anti-records. Therefore, there are no records that are not anti-records. Hence,

$$erec(\omega_1) = 0. \qquad \square$$

Proposition 3.4

Let $\omega_1 \in G_p^{\Gamma_1}$. Then the

$$rar(\omega_1) = p.$$

Proof:

By the Definition of record-antirecord. Record-Antirecord records are records that are both record and anti-records. By Proposition 3.1 and 3.2, all the elements in ω_1 are both records and anti-records. Hence,

$$rar(\omega_1) = p. \quad \square$$

Proposition 3.5

Let $\omega_1 \in G_p^{\Gamma_1}$. Also let $\Omega = \{rec, arec, erec, rar\}$ then the

$$\sum_{\mu \in \Omega} \mu(\omega_1) = 3p.$$

Proof:

By proposition 3.1. 3.3, 3.3 and 3.4, the $rec(\omega_1), arec(\omega_1), erec(\omega_1)$ and $rar(\omega_1)$ are $p, p, 0$ and p respectively, Therefore, the

$$\sum_{\mu \in \Omega} \mu(\omega_1) = 3p. \quad \square$$

Proposition 3.6

Let $\omega_{p-1} \in G_p^{\Gamma_1}$. Then the

$$rec(\omega_{p-1}) = 2.$$

Proof:

For any $G_p^{\Gamma_1}$, $\omega_{p-1} = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p-1 & p \\ 1 & p & p-1 & . & . & . & 3 & 2 \end{pmatrix}$. Therefore the only pair $(j, \omega_{p-1}(j))$ for which $\omega_{p-1}(k) < \omega_{p-1}(j)$ for all $k < j$ are 1 and p. Therefore,

$$rec(\omega_{p-1}) = |\{1, p\}| = 2. \quad \square$$

Proposition 3.7

Let $\omega_{p-1} \in G_p^{\Gamma_1}$. Then the

$$arec(\omega_{p-1}) = 2.$$

Proof:

For any $G_p^{\Gamma_1}$, $\omega_{p-1} = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p-1 & p \\ 1 & p & p-1 & . & . & . & 3 & 2 \end{pmatrix}$. Therefore the only pair $\left(j, \omega_{p-1}(j)\right)$ for which $\omega_{p-1}(k) > \omega_{p-1}(j)$ for all $k > j$ are 1 and 2. Therefore,

$$arec(\omega_{p-1}) = \left|\{1, 2\}\right| = 2.$$

Proposition 3.8

Let $\omega_{p-1} \in G_p^{\Gamma_1}$. Then the

$$erec(\omega_{p-1}) = 1.$$

Proof:

By the Definition of exclusive record. Exclusive records are records that are not anti-records. By Proposition 3.6 and 3.7, the $rec(\omega_{p-1}) = \left|\{1, p\}\right|$ and the $arec(\omega_{p-1}) = \left|\{1, 2\}\right|$. Therefore, the only record that is not anti-record is p. Hence,

$$erec(\omega_{p-1}) = \left|\{ p\}\right| = 1.$$

Proposition 3.9

Let $\omega_{p-1} \in G_p^{\Gamma_1}$. Then the

$$rar(\omega_{p-1}) = 1.$$

Proof:

By the Definition of exclusive record. Exclusive records are records that are not anti-records.

By Proposition 3.6 and 3.7, the $rec(\omega_{p-1}) = |\{1, p\}|$ and the $arec(\omega_{p-1}) = |\{1, 2\}|$.Therefore, the only record that is also anti-record is 1. Hence,

$$rar(\omega_{p-1}) = |\{1\}| = 1 .$$

4. CONCLUSION

In this paper, we compute the record, antirecord, exclusive record, and record-antirecord on Γ_1-non deranged permutations. We discovered that the rec of ω_{p-1} is equal to the arec of ω_{p-1}. Similarly, the erec of ω_{p-1} is equal to the rar of ω_{p-1}.We also devise a straightforward technique for determining the record, anti-record, exclusive record and record anti record of any ω_1 in Γ_1-non deranged permutation group $G_p^{\Gamma_1}$.

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