ENHANCEMENT OF SPEED CONTROL FOR BRUSHLESS DC MOTOR USING PID CONTROLLERS

Lawal Muhammad Isah and Musa Umar

Department of Electrical Engineering Technology Federal Polytechnic Kaura Namoda, Zamfara State <u>dangoggo@gmail.com</u>

ABSTRACT

In this Paper, a speed control system for a Brushless DC (BLDC) motor is to be analysed and enhanced to increase the efficiency of the motor. The objective is to keep the magnetic field between stator and rotor always perpendicular in order to generate the maximum torque with the minimum consumption of energy. The first step taken in this paper is the analysis of the model of a BLDC motor. By using the set of parameters given from a book (Analysis of Electric Machinery), a dynamic model is programmed in Matlab to perform all the experiments required. PI controller is implemented in order the speed tracking to be achieved. The manipulated variable is the input voltage and the controlled variable is the speed.

1. INTRODUCTION

Brushless Direct Current (BLDC) motors are synchronous electric motors, commutated electronically and are rapidly gaining popularity due to several advantages over brushed DC and induction motors, which have mechanical commutators and brushes. Some of the advantages that BLDC motors have compared to conventional dc motors are stated below (Krause, Paul, Wasynczuk, Oleg, Sudhoff, Scott, 2000):

- High efficiency
- Better speed versus torque characteristics
- · Reliability
- Long lifetime (no brushed and commutator erosion)
- Noiseless operation
- Higher speed ranges

These advantages make BLDC motors to be used in many industrial applications in Instrumentation, Appliances, Factory automation Equipment, Military, Automotive, Aerospace and Medical fields, as well as in Industrial Automation equipment.

The construction of a BLDC motor is very similar to AC motors and it is known as the permanent magnet synchronous motor. In Fig1 the structure of a BLDC motor is illustrated. The BLDC motor consists of the stator (in our model the induced part of the machine, 3-phase windings) and the rotor (inductor of the machine) which is a permanent magnet. Commutation refers to the process which converts the input direct current to alternating current and properly distributes it to each winding in the armature (Efstratios N. Pistikopoulos, Vivek Dua, Nikolaos A. Bozinis, Albert Bemporad, Manfred, 2000). In BLDC motors, this process is done by semiconductor devices such as transistors as there are no brushes and commutator.



Figure 1: 3phase Brushless DC Motor (P. Krause, 1986)

The BLDC motor is a synchronous motor and its principle of operation is based on the fact that, when current enters into the windings, torque is produced because of the interaction between the magnetic field, generated by the stator and the magnetic field, generated by the rotor (permanent magnets). These magnetic fields are rotating at the same frequency, producing a torque, which makes the motor rotate. The main objective is to keep these two magnetic fields perpendicular to each other, in order to achieve maximum torque and to minimize the consumption of energy.

2. REAL MODEL MATHEMATICAL ANALYSIS

In this section, the differential equations that represent the real model's dynamics are derived. Furthermore, the formula of the manipulated input voltage for the speed control is derived in matrix form. In addition to this, the torque equations needed to be maximised for the enhancement problem, is also obtained. The theoretical background from Krause (Krause, Paul C.; Wasynczuk, Oleg; Sudhoff, Scott, 2002) is used, for the establishment of the voltage and torque equations.

2.1 Park's Transformation and flux linkage

For the simulation and the analysis of the BLDC motor the equivalent circuit in Fig. 2 is used. The stator windings are identical windings displaced 120°, each with N_s equivalent turns and resistance r_s .



Figure 2: Equivalent Circuit of 3-phase brushless DC machine

According to this transformation, the variable of stationary circuit elements of figure 2 are transformed to an arbitrary reference frame expressed as:

Where the vector of variables is expressed in the following way:

Where, f is a variable of voltage or current. The Park transformation is defined:

$$\overline{K_s} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \dots \dots \dots \dots (4)$$

The inverse is given by:

$$\overline{K_s}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1\\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1\\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$
.....(5)

The angular velocity ω and the angular position θ of the arbitrary reference frame are related by $d\theta$ \sim

$$\omega = \frac{dt}{dt}$$
(6)

The flux linkages are expressed as

Where, $\overline{\lambda_m}$ is the residual magnetic field of the permanent magnets in respect to the stator angle θ_r . The inductance expressed in matrix form is presented below:

$$\overline{L_{s}} = \begin{bmatrix} L_{is} + L_{A} - L_{B}\cos 2\theta_{r} & -\frac{1}{2}L_{A} - L_{B}\cos 2\left(\theta_{r} - \frac{\pi}{3}\right) & -\frac{1}{2}L_{A} - L_{B}\cos 2\left(\theta_{r} + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_{A} - L_{B}\cos 2\left(\theta_{r} - \frac{\pi}{3}\right) & L_{is} + L_{A} - L_{B}\cos 2\left(\theta_{r} - \frac{2\pi}{3}\right) & -\frac{1}{2}L_{A} - L_{B}\cos 2\left(\theta_{r} + \pi\right) \\ -\frac{1}{2}L_{A} - L_{B}\cos 2\left(\theta_{r} + \frac{\pi}{3}\right) & -\frac{1}{2}L_{A} - L_{B}\cos 2\left(\theta_{r} + \pi\right) & L_{is} + L_{A} - L_{B}\cos 2\left(\theta_{r} + \frac{\pi}{3}\right) \end{bmatrix}$$
(8)

Where, L_{is} is the auto inductance in stator and θ_r is the angle of the rotor with respect to the stator. The mutual inductances can be expressed using the geometric properties of the machine as the following expressions:

$$L_{A} = \left(\frac{N_{s}}{2}\right)^{2} \pi \mu_{0} r l \alpha_{1} \dots (9)$$

$$L_{B} = \frac{1}{2} \left(\frac{N_{s}}{2}\right)^{2} \pi \mu_{0} r l \alpha_{2} \dots (10)$$

 N_s is the number of turns in coil in stator, r is the radius of the average gap circle and l is the deep of the motor (Martin M. Gonzalez, 2009).

Torque and voltage equations in machine variables 2.2 The voltage equations are expressed below:

By substituting equation (4) into (11) and the product of this substitution into (13), we take:

THE BEAM: JOURNAL OF ARTS & SCIENCE, VOL. 9, (2016) ISSN: 1118-5953

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + w \begin{pmatrix} 0 & \frac{3}{2}L_A + \frac{3}{2}L_B - L_{is} & 0 & 0 \\ -\frac{3}{2}L_A + \frac{3}{2}L_B - L_{is} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + \begin{bmatrix} \lambda_{m'} \\ 0 \\ 0 \end{bmatrix} \end{pmatrix}$$
$$+ p \begin{pmatrix} \left[\frac{3}{2}L_A - \frac{3}{2}L_B + L_{is} & 0 & 0 \\ 0 & \frac{3}{2}L_A + \frac{3}{2}L_B + L_{is} & 0 \\ 0 & 0 & L_{is} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + \lambda_{m'} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{pmatrix}$$

.(14)

The above system of equations can be presented in the following equivalent equations:

$$v_{qs} = r_{s}i_{qs} + \omega \left(\frac{3}{2}L_{A} + \frac{3}{2}L_{B} + L_{is}\right)i_{ds} + \omega\lambda_{m}' + p\left(\frac{3}{2}L_{A} - \frac{3}{2}L_{B} + L_{is}\right)i_{qs}$$

$$v_{ds} = r_{s}i_{ds} + \omega \left(-\frac{3}{2}L_{A} + \frac{3}{2}L_{B} - L_{is}\right)i_{qs} + p\left(\frac{3}{2}L_{A} + \frac{3}{2}L_{B} + L_{is}\right)i_{ds} \dots (15)$$

$$v_{0s} = r_{s}i_{0s} + pL_{is}i_{0s}$$

The electrical torque is described in the following equation [2]:

$$T_{e} = \left(\frac{P}{2}\right) \left\{ \begin{array}{l} \frac{L_{md} - L_{mq}}{3} \left[\left(i_{as}^{2} - \frac{1}{2}i_{bs}^{2} - \frac{1}{2}i_{cs}^{2} - i_{as}i_{bs} - i_{as}i_{cs} + 2i_{bs}i_{cs}\right)\sin 2\theta_{r} \right] \\ + \frac{L_{md} - L_{mq}}{3} \left[\frac{\sqrt{3}}{2} \left(i_{bs}^{2} + i_{cs}^{2} - 2i_{as}i_{bs} + 2i_{as}i_{cs}\right)\cos 2\theta_{r} \right] \\ + \lambda_{m'} \left[\left(i_{as} - \frac{1}{2}i_{bs} - \frac{1}{2}i_{cs}\right)\cos \theta_{r} + \frac{\sqrt{3}}{2} \left(i_{bs} - i_{cs}\right)\sin \theta_{r} \right] \end{array} \right\} \dots \dots (16)$$

Where the inductances are expressed using the mutual inductances:

$$L_{mq} = \frac{3}{2} (L_A - L_B)....(17)$$

$$L_{md} = \frac{3}{2} (L_A + L_B)....(18)$$

The mechanical torque generated is expressed below in terms of rotor inertia moment J and torque load $T_{L}[2]:$

The next step is to convert the electrical torque in (19) into a new mobile reference framework which contains the current variables i_{gdos}

This can be done by using the park's transformation:

$$\overline{i_{abcs}} = \overline{K_s}^{-1} \overline{i_{qd0s}} = \begin{bmatrix} \cos\theta & \sin\theta & 1\\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1\\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{ds} \end{bmatrix} = \begin{bmatrix} i_{qs}\cos\theta + i_{ds}\sin\theta + i_{0s} \\ i_{qs}\cos\theta + \frac{2\pi}{3} + i_{ds}\sin\theta + i_{0s} \\ i_{qs}\cos\theta + \frac{2\pi}{3} + i_{ds}\sin\theta + i_{0s} \end{bmatrix} \dots$$

(20)

Also, by Gonzalez (Martin M. Gonzalez, 2009):

As a result, the equation that describes the torque is derived by presenting both electrical and mechanical torque as one equation and it is expressed below:

By rearranging terms and substituting the first derivative into p, the equations (15) and (22) are becoming the differential equations that describe the dynamics of the system. These equations are the following:

$$\frac{di_{qs}}{dt} = \frac{1}{\left(\frac{3}{2}L_{A} - \frac{3}{2}L_{B} + L_{is}\right)} \left(v_{qs} - r_{s}i_{qs} - \omega\left(\frac{3}{2}L_{A} + \frac{3}{2}L_{B} + L_{is}\right)i_{ds} - \omega\lambda_{m}'\right)$$

$$\frac{di_{ds}}{dt} = \frac{1}{\left(\frac{3}{2}L_{A} + \frac{3}{2}L_{B} + L_{is}\right)} \left(v_{ds} - r_{s}i_{ds} - \omega\left(-\frac{3}{2}L_{A} + \frac{3}{2}L_{B} - L_{is}\right)i_{qs}\right) \dots (23)$$

$$\frac{di_{0s}}{dt} = \frac{1}{L_{is}} \left(v_{0s} - r_{s}i_{0s}\right)$$

$$\frac{d\omega}{dt} = \frac{1}{J} \left(\frac{3}{2}\left(\frac{P}{2}\right)^{2} \left(3L_{B}i_{qs}i_{ds} + \lambda_{m}'i_{qs}\right) - B_{m}\omega - \left(\frac{2}{P}\right)T_{L}\right)$$

Inductance transformation (Krause, Paul C.; Wasynczuk, Oleg; Sudhoff, Scott, 2002):

$$L_{q} = \frac{3}{2}L_{A} - \frac{3}{2}L_{B} + L_{is}$$

$$L_{d} = \frac{3}{2}L_{A} + \frac{3}{2}L_{B} + L_{is}$$
(24)

Equations (23) take the following form, after substituting (24) into (23) and assuming that all the input signals are balanced so that the zero current circuit can be removed (Martin M. Gonzalez, 2009).

Equation (25) is written in vector matrix form as:

$$\begin{bmatrix} \frac{d_{iqs}}{dt} \\ \frac{d_{ids}}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{r_s}{L_q} & 0 & 0 \\ 0 & -\frac{r_s}{L_d} & 0 \\ \frac{3}{2} \left(\frac{P}{2}\right)^2 \lambda_m & 0 & -\frac{B_m}{J} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ \omega \end{bmatrix} + \begin{bmatrix} -\frac{L_d}{L_q} \omega i_{ds} \\ \frac{L_q}{L_d} \omega i_{qs} \\ \frac{3}{2} \left(\frac{P}{2}\right)^2 (L_d - L_q) i_{ds} i_{qs} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_q} & 0 & 0 \\ 0 & \frac{1}{L_d} & 0 \\ 0 & 0 & -\frac{2}{JP} \end{bmatrix} \begin{bmatrix} V_q \\ V_d \\ T_L \end{bmatrix}$$

Equation (26) is the state space equation that describes the behaviour of the motor. The system is considered to be non linear due to the second term of the equation.

2.3 Control approach for speed reference tracking by PI controller

Speed control is achieved by changing the voltage reference of the motor. The optimum speed loop maintains the motor at its target speed. The 3-phase input voltages V_a, V_b, V_c are sinusoidal and synchronised with a phase difference of 2pi/3 and the same frequency equal to 1. The input voltage vector is illustrated below:



Figure 3: 3-phase input voltage in abc reference framework

The motor cannot work in open loop so we need at least the inner close loop to switch the voltages on each phase. Each phase depends on the angle alpha between the electromagnets and the position of the motor.

Speed controller calculates the difference between the reference speed and the actual speed producing an error, which is fed to the PI controller. PI controllers are used widely for motion control systems. They consist of a proportional gain that produces an output proportional to the input error and an integration to make the steady state error zero for a step change in the input.

The implementation of the speed controller requires the linear model of the BLDC motor around an operation point.

The linear system is described as SISO in state space form:

$$\dot{x} = Ax + Bu$$

$$\begin{aligned} x &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$
(28)

The input is the amplitude voltage u and the output is the speed.

$$A = \begin{bmatrix} -\frac{r_s}{L_q} & -\frac{w_0}{L_q} & -\frac{L_d i_{ds0}}{L_q} - \frac{\lambda_m}{L_q} \\ \frac{\omega_0 L_q}{L_d} & -\frac{r_s}{L_d} & \frac{L_q i_{qs0}}{L_d} \\ \frac{3}{2} \left(\frac{P}{2}\right)^2 \frac{(L_d - L_q)}{J} i_{ds0} + \frac{3}{2} \left(\frac{P}{2}\right)^2 \frac{\lambda_m}{J} & \frac{3}{2} \left(\frac{P}{2}\right)^2 \frac{(L_d - L_q)}{J} i_{qs0} & -\frac{B_m}{J} \end{bmatrix}$$
$$B = \begin{bmatrix} \frac{1}{L_q} \frac{\cos(a_0)}{\sqrt{3}} \\ \frac{1}{L_d} \frac{\sin(a_0)}{\sqrt{3}} \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$
$$D = 0$$

The transfer function of the motor equation is obtained by: $G = C(sI - A)^{-1}B$ (31)

After obtaining the system in state space form, the transfer function of the closed loop system has to be calculated. The transfer function of the PI controller is:

$$C_{tf} = k_i \frac{\tau_i s + 1}{\tau_i s} \dots \tag{32}$$



Fig. 4: Closed loop motor system with PI controller

3. **RESULTS**

3.1 Motor Model under no Controller

The objective is to calculate efficiency of the motor without a speed controller. The experiment was done under torque load equal to 0.1Nm and then the torque load was increased to 0.4Nm. The efficiency in different torque loads is calculated in figure 6. The actual speed of the motor in different torque loads is also obtained in figure 7.



Figure 6: Efficiency at TL=0.1Nm and TL=0.4Nm



The next step is to eliminate the error between the actual speed and the reference speed. In the above experiments the nominal speed was 200rad/sec and the approach to eliminate this error is to implement a PI controller as described in the following experiment.

3.2 Model Under PI Controller

The control requirements are the same as described in Gonzalez (Martin M. Gonzalez 2009):

- Settling time: <1 sec
- Maximum Overshoot: < 10%
- Maximum Efficiency Possible: 100%
- Nominal Voltage: 22.5V
- Maximum Quadrature Voltage: 15.9V
- Reference Speed (Half Nominal): 100rad/sec
- Torque Load: 0.1Nm

The brushless machine is a non linear model and in order to implement the PI controller it is vital to linearize it at some operation points. The model described by differential equations in (26) is linearized using Taylor series and the SISO linearised model is derived in state space form. The operating conditions are the following:

$$I_{a0} = 0.1A$$

$$I_{d0} = 0.01A$$

$$W_0 = 74.21 rad / sec$$

The system is third order with step response presented on figure 5.





The system has the following poles, zeros and steady state gain: Zeros of SISO: -281.2447 Poles of SISO: -277.79 + 114.55i -277.79 - 114.55i -166.97

Steady State Gain: 20.8



Figure10: Input Voltage with PI controller at TL=0.4Nm

In figures 9 and 10, the output current for each change of the torque load is presented. After the increase in torque, the output current is being slightly decreased due to the additional effort of the motor to handle the extra torque load.



Figure11: Current output with PI controller at TL=0.1Nm



Figure12: Current output with PI controller at TL=0.4Nm

10 UMARU ALI SHINKAFI POLYTECHNIC SOKOTO, NIGERIA In the above figure, the speed response is presented for different torque loads. As it is shown, speed reference tracking is achieved, with settling time smaller than 1 sec and no overshoot bigger than 10%. The requirements after implementing the PI controller have been fulfilled. The response of the system under TL=0.1Nm is slightly faster than the response under TL=0.4Nm.



Figure 14: Efficiency with PI controller at TL=0.1Nm and TL=0.4Nm

The efficiency of the motor is illustrated under different values of torque load. It is observed that a better efficiency is achieved with a smaller value of torque load.

4. CONCLUSIONS

The model used for this work is a brushless synchronous asymmetric motor with sinusoidal three phase input voltage. The mathematical model is considered to be single input single output (SISO), the manipulated variable is the input voltage and the control variable is the speed. The motor is described by non linear equations, so for the implementation of the PI controller for the speed tracking reference the system has to be linearised at some operation points. After linearization, the PI controller is implemented to the linear system and the response of the motor to different values of torque load and speed references is obtained and the efficiency of the motor is compared under different torques.

Experiments show that the speed reference tracking is easy to be achieved and the PI controller fulfils the requirements of settling time equal to less than 1 sec and overshoot equal to less than 10%. This makes sense, as with a bigger value of torque load, there is more power consumption.

5. **REFERENCES**

Martin Maya Gonzalez,(2009). Optimization of speed control of BLDC motor, Dissertation Project University of Manchester.

- Krause, Paul C.; Wasynczuk, Oleg; Sudhoff, Scott,(2002). Analysis of electric machinery and drive systems, second edition, United States of America, IEEE Press, Wiley-Interscience.
- Efstratios N. Pistikopoulos, Vivek Dua, Nikolaos A. Bozinis, Albert Bemporad, Manfred Morari, (2000). On-line optimization via off-line parametric optimization tools. *Elsevier*, vol.24, pp 183-188.

- Gennady, G. Kulikov, Haydn A. Thompson, (2004). *Dynamic Modelling of Gas Turbines*. London: Springer-Verlag.
- Microcontroller Division Applications,(2000). An introduction to sensorless DC motor applications,<u>http://www.st.com</u>
- Anand, D.K. (1984). *Introduction to control systems*, 2nd Edition, Pergamon press ltd., Headington Hill Hall, oxford OX3, OBW, England.
- Seborg, D.E., Thomas F. Edgar, Duncan A. Mellichamp (1989). Process Dynamics and Control, John Wiley & Sons.
- Skogestad, S. and Ian Postlethwaite, (2005). *Multivariable Feedback Control (Analysis and Design),* Second edition. John Wiley & Sons.
- P. Pillay and R. Krishnan. (1989). Modelling, simulation, and analysis of permanent-magnet motor drives. The permanent-magnet synchronous motor drive, IndustryApplications. IEEE Transactions on, 25, 265-273.
- P. Krause (1986). Analysis of Electric Machinery. New York: McGraw-Hi11.