LINEAR PROGRAMMING: A TOOL FOR EFFICIENT COST-MINIMIZATION ON POULTRY-FEEDS

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ABSTRACT
The emergence of Linear programming in the late 1940s was the result of successful military application during World War II. It provided answers on how to minimize cost (or maximize any desired function) whilst achieving a given set of objectives. This article aimed at minimization of cost of poultry feeds. The data used was collected using both primary and secondary sources. The data obtained was formulated into linear programming problem using simplex algorithm with optimal solution obtained after the (4th) iteration. Here layers and broilers feed are in the ratio $\left(\frac{8}{9} \textit{ and } \frac{10}{9}\right)$ *respectively, having a minimum cost of producing a tone as (₦468, 995.6k). The study shows that for an efficient and effective production of poultry feeds, determination of minimum cost of production using the linear programming approach is a necessity.*

Keywords: Linear Programming, Simplex Method, Poultry Feeds, Cost Minimization

1. INTRODUCTION

A key problem faced by managers is how to allocate scarce resources among activities or projects. Linear programming, or LP, is a method of allocating resources in an optional way. It is one of the most widely used operations research (OR) tools. It has been used successfully as a decision making aid in almost all industries, in financial and service organizations.

Programming refers to mathematical programming. In this context, it refers to a planning process that allocates resources – labour, materials, machines, and capital – in the best possible (optimal) way so that costs are minimized or profits are maximized. In LP, these resources are known as decisions variables. The criterion for selecting the best values of the decision variables (e.g., to maximize profits or minimize costs) is known as the objectives function. The limitations on resources availability form what is known as a constraint set.

The business of rearing livestock especially poultry is cost-sensitive. Feed cost, for instance, account for between 65% and 70% of the total cost of raising poultry (Bamiro, 2001). This and other cost of poultry production has increased the price of eggs and other poultry product beyond the reach of most Nigerians. The economic implication on the part of the producer is that any producer who can lower his costs by a few Naira per crate of eggs will gain a larger share of the market. The desired cost-saving can be achieved via vertical integration (Akinwumi, 1976 and Queden, 1996).

Dependence on the use of the external market to obtain an input or to exchange an output may be through the use of a contract or a spot market. The quality of the input and the timeliness of the supply cannot be guaranteed. The failure of the external market creates profit and risk incentives for the farm to integrate vertically (kilmer, 1986).

Feed, the major factor militating against the poultry industry, hamper production, not only on the basis of high cost but also due to low quality feeds supplied by the feed millers which has a negative impact on the productivity – low level of eggs production as well as rendering the birds susceptible to disease, hence the need for backward vertical integration via the production of quality feeds by each poultry farm – firm (Bamiro, 2001). In recent times the experience of farmers have shown that poultry production has been suffering some setbacks caused by increasing cost of feeds among others, this reduce the net return from the business significantly (Aihonsu, 1999). Also many of the existing poultry farms are folding up and prospective investors are becoming increasingly reluctant to invest (Aihonsu, 1999).

This situation threatens the survival of poultry industry and these calls for concerted efforts to save the industry from total collapse. Failure to do this could lead to a serious reduction in poultry production and protein intake of people resulting into malnutrition and ill health, which again will transform into lower productivity and output. There is, therefore, the problem of finding adequate means of increasing net returns to farmers in the poultry business. The net returns must be significant enough to retain the farmers in the business and attract more participant.

Given the fact that the farmer has little or no control over the demand and prices of the product, because of the nature of the market which is more or less a perfectly competitive, a more plausible approach to increasing net return to farmers is to reduce the cost of production (Aihonsu, 1999). On the basis of the foregoing, poultry farmers need to seek for means to reduce costs, risks and thus increase the profitability of the poultry enterprise. According to (Buzzel, 1983) and (Ouden, 1996) the major objective of vertical integration is to eliminate or at least reduce, the transaction costs incurred when separate companies own two stages of production.

2. HISTORICAL BACKGROUND OF POULTRY IN NIGERIA

Poultry industry is an emerging agri-business and has established its position as the fastest growing segment in the agricultural sector in Nigeria. With increased acceptance of chicken egg and meat, the demand for this product is ever increasing (Shrestha, 2003). Poultry sector has tremendous employment potential and would go a long way in reducing unemployment problems in Nigeria. The growth of the industry is influenced by many factors, critical among which is the quality of day-old-chicken supplied to farmers. To succeed in poultry farming, chick quality assessment is most important for the poultry farmers as seed quality is to crop production. However, this important factor appears to be neglected in Nigeria, as no agency is charged with the responsibility of assessing the quality of day-old-chicks supplied to farmers.

The Nigerian poultry industry is characterized by few hatcheries (connected mainly in some part of the country) and many small holder farmers (scattered across the nation). The farmers place their orders day-old-chicks from these hatcheries through agents and distributors across the country. Consequently, the demand for day-old-chicks is by far more than its supply in most parts of the country, sometimes order take as long as a month or more before they are supplied especially during peak periods of demand. Desperation on the part of the farmers and agents result in sourcing and supply of chicks from unknown and/or of questionable medical history. Some of the agents even supply chicks in unlabeled boxes, which sometimes contain many dead chicks. Such situations are expectedly characterized by heaving brooder losses at the expenses of the farmers. Occasionally, when complaints become overwhelming, farmers are compensated. Ideally, farmers that buy from accredited stocks are often assured of freedom from all hatchery bal seed infections like poultry disease and fowl typhoid. When chicks are brought a day old, mortality should not exceed 5% requires an investigation.

3. OBJECTIVES OF THE STUDY

The objectives of this study is to optimize profit derived from the production of poultry feeds.
Specific objective, therefore, are:-

1. To determine how linear programming can be used in solving physical problems.
2. To analyze some resources available to the company for producing poultry feeds.
3. To determine optimal utilization of available resources.
4. To propose the best food-mix to be produced by the company that yields maximum profit and also find the least requirement of material used in production that gives the best result at minimum cost of production.
5. To make recommendation of the items to be produced most, in order to maximize profit and those to be produced least, to minimize cost of production through utilization of limited available resources.

4. SCOPE AND LMITATION

The scope of the study is associated with data collected from the S.G/Adiya farms, based on the result, to give recommendation of items to be produced most to utilize available resources and maximize profit. The following are the items that S.G/Adiya farms produced:-

1. Poultry chickens
2. Poultry eggs.

This article is limited to model of linear programming problem and it's solution by means of one, model; (simplex method) using the data collected from the S/Adiya farms, which is the case study.

5. METHODOLOGY

In this study, data was collected from S.G/Adiya farms. This is followed by the extensive analysis of data which was carried out to define all available items, so as to determine the quantity to optimize in a form that is more convenience for analysis. Secondly, identifying all requirements, limitations and expressing them mathematically in terms of decision variables. Finally, the mathematical model constraint is solve, by using the simplex method in order to know the value of decision variables that yields the optimal profit as well as the least cost of production.

6. LINEAR PROGRAMMING TECHNIQUES AND ALGORITHM USED

In many physical problems the graphical methods for solving the programming problem appears to be extremely tedious and difficult if the number of variables and constraints are large. In this regard the most widely and efficient techniques used in solving linear programming problem is simplex methods. This method was originated by an American mathematician George B. Dantzig in 1947.

Simplex method as a general purpose linear programming algorithm is widely used to solve large scale problem and it is one of the existing method of solving LLPs which has a wider applicability and computational efficiency. Its efficiency cannot be compared to any algorithm (except on special types of problem), Abdullahi (2006). The simplex technique involves a series of iterations. Successive improvement are made until an optimum solution is achieved. The method is based on simple ideas related to the solution of simultaneous linear algebraic equation using what is known as the Gauss Jordan Elimination method.

The algorithm gives either the exact solution in a finite number of steps or no solution with an indication that there is an unbounded solution. The algorithm starts from a basic feasible solution and successfully iterates until a feasible solution at optimality is reached. The revised simplex method known as primal simplex method, Taha (2005) is the same as simplex method but put in Matrix form and it has not meet wide acceptance for hand computation Hadley (1962) but it has some real advantage over the simplex method when a digital computer is to be used. Most of the existing software such as Lindo, lingo, Tora, Ampl, Excel solver etc, are coded according the computations of the revised simplex method.

The simplex technique involves generating series of solution with a process in systematic steps from initial basic feasible solution to other basic feasible solutions and finally in a finite number of steps, to an optimal basic feasible solution. The objectives function (value) at each step is appreciable than that at the preceding step.

Suppose we want to maximize an existing or a given linear programming problem, then the standard maximization problem of linear programming and the corresponding initial simplex tableau will be written as follows:

$$Max. \qquad Z = \sum_{j=1}^{n} C_j X_j$$

(2.5)

Subject to

$$\sum_{j=1}^{n} a_{ij} X_j \ (\leq, \geq = \mathrm{b}_i,\ i = 1, 2, 3\ldots\ m.)$$

(2.6)

Where

$x_i \geq 0$ for $i = 1,2,3,\ldots,n$

$b_i \geq 0$ for $i = 1,2,3,\ldots,m$

To solve the above LP problem we need to introduce the slack variables $S_i \geq 0$ for i = 1, 2, 3,…, m introducing the slack variables the objective function and the constraint of the LP problem above are changed to read.

$$\text{Max. } Z = \sum_{j=1}^{n} C_j X_j + 0x_j$$

(2.7)

Subject to

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n + 1S_1 + 0S_2 + 0S_3 + \ldots + 0S_m = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n + 1S_1 + 0S_2 + 0S_3 + \ldots + 0S_m = b_1$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n + 1S_1 + 0S_2 + 0S_3 + \ldots + 0S_m = b_3$$

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$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n + 1S_1 + 0S_2 + 0S_3 + \ldots + 0S_m = b_m$$

(2.8)

The two basic conditions have to be followed when representing the algebraic solution of LP.

1. The right hand side of all the constraints (with the exception of the non-negativity restriction) are to be express as non-negative, if there exist, then right hand side of the equations are to be multiplied by -1 so as to change the negative right hand side to positive.
2. All the variables are non-negative.

- The difference between the right hand side and left-hand side in (≤) constraints, yields an unused or slack amount. Therefore to convert the inequality (≤) to an equation, a non-negative slack variable is added to the left hand side of the constraints.
- Normally (≥) gives a lower limit on the activity of linear programming model. In view of this, the amount by which the left hand side exceed the minimum limit represent a surplus. To avert (≥) inequality constraints to (=) a non-negative surplus variables is subtracted from the left hand side of the inequality.

The process encompasses the following steps algorithms:
Step 1: Determining a starting basic feasible solution and set up the initial tableau.
Step 2: Develop a revise tableau using the information contain in the first tableau.
Step 3: inspect to see if it is optimal, this is done by selecting an entering variable using the optimality conditions. The process continues until when there is no entering variable, and last solution is optimal. Optimality condition states that the entering variable (i.e the non-basic variable that will enter the set of basic variable) in a maximization (Minimization) problem is the non-basic variable having the most negative (positive) coefficient in the Z – row. (Abdullahi 2006).
Step 4: Select a leaving variable using the feasible condition in both maximization and minimization problems, the leaving variable is a basic variable associated with the smallest ratio (with strictly positive a denomination) which is carried out by dividing the right hand side of the constraints by the co-efficient under the entering variable.
Step 5: Repeat the steps until no further iteration is possible then later determine the new basic solution by using the appropriate Gauss Jordan Computation.

The Gauss – Jordan (row operations) computations relates the pivot column and the pivot row with the entry and leaving variables respectively. The pivot element is at the point of interaction where the pivot column and pivot rows coincide. In Gauss Jordan computations the following two steps are needed to produce the new basic solution.

- Pivot row

  New pivot row = Current row ÷ pivot element
- All rows, including Z

  New row = (current row) – (the pivot column coefficient) x (New pivot row).

A convenient way to write the information contained in the problem is define in tabular form called initial simplex tableau.
In order to ensure minimization in cost of poultry feeds, the following data were collected.

TABLE 1: The proportion of the ingredient required to make a Tonne of Finished layers feeds

INGREDIENT	C/O PER TONNE	QUANTITY (kg)	COST(₦)
Soya beans	60%	600kg	168,600
Blood meal	20%	200kg	51,200
Salt	0.5%	5kg	720
Vitamin mix	9.5%	95kg	19,000
Bone meal	10%	100kg	12,000
			Total= ₦251,520

TABLE 2: The proportion of ingredient required to make a tonne of finished broilers feeds

INGREDIENT	(%) PER TONNE	QUANTITY (kg)	COST (₦)
Soya beans	20	200	56,200
Blood meal	11	110	28,160
Salt	0.5	5	720
Vitamin mix	95	95	19,000
Bone meal	19	190	228,000
Maize	40	400	9,400
			Total = ₦341,480

TABLE 3: Available ingredient to produce a tonne each of both layers and broilers feeds

INGREDIENT	MAXIMUM AVAILABLE	COST (₦)
Soya beans	1000kg	28,1000
Blood meal	1000kg	8,533.30
Salt	10kg	1,440

Vitamin mix Bone meal Maize	250kg 300kg 500kg	500,000 36,000 117,500
		Total = ₦571,273.30

TABLE 4: The quantity of finished ingredient required to make feed of layers and broilers

INGREDIENT	LAYERS	BROILERS	THE AVAILABLE
Soya beans	600kg	200kg	1000kg
Blood meal	200kg	110kg	1000/3kg
Salt	5kg	5kg	10kg
Vitamin mix	95kg	95kg	250kg
Bone meal	100kg	190kg	300kg
Maize	-	400kg	500kg
Cost(₦)	**251,520**	**220,880**	**571,273.30**

The data collected for this research was from primary sources. The type of ingredients which made up the ratio is an attracted ingredient which is determined through the market survey. Also, the officers in charge of the poultry were interviewed on the ways and proportion with which the ingredients are being mixed. The data used for this research was obtained from the S.G/Adiya farms, along Bodinga road – Sokoto.

7. DATA ANALYSIS

From Table: 4

Let layers feeds = x_1, let broiler feed = x_2

The objective function:-

Min Z = 251520 x_1 + 220880 x_2

The constraints

For Soya beans; $600x_1 + 200x_2 \leq 1000$

For Blood meal; $200x_1 + 110x_2 \leq \frac{1000}{3}$

For Salt; $5x_1 + 5x_2 = 10$

For Vitamin mix; $95x_1 + 95x_2 \leq 250$

For Bone meal; $100x_1 + 190x_2 \leq 300$

For Maize; $0x_1 + 400x_2 \leq 500$

$x_1 + x_2 \geq 0$

The linear programming model is:-

Min Z = 251,520 x_1 + 220880 x_2

Subjected to

$$600x_1 + 200x_2 \leq 1000$$

$$200x_1 + 110x_2 \leq \frac{1000}{3}$$

$$5x_1 + 5x_2 = 10$$

$$95x_1 + 95x_2 \leq 250$$

$$100x_1 + 190x_2 \leq 300$$

$$0x_1 + 400x_2 \leq 500, \qquad x_1, x_2 \geq 0$$

By adding the slack variables to change the inequality to equalities the equations become:-

Min Z = $251,520x_1 + 220880x_2 + OS_1 + OS_2 + OS_3 + OS_4 + OS_5 + OS_6$

s.t

$$600x_1 + 200x_2 + 1S_1 + OS_2 + OS_3 + OS_4 + OS_5 + OS_6 = 1000$$

$$200x_1 + 110x_2 + OS_1 + 1S_2 + OS_3 + OS_4 + OS_5 + OS_6 = \frac{1000}{3}$$

$$5x_1 + 5x_2 + OS_1 + OS_2 + OS_3 + OS_4 + OS_5 + 1S_6 = 10$$

$$95x_1 + 95x_2 + OS_1 + OS_2 + 1S_3 + OS_4 + OS_5 + OS_6 = 250$$

$$100x_1 + 190x_2 + OS_1 + OS_2 + OS_3 + 1S_4 + OS_5 + OS_6 = 300$$

$$0x_1 + 400x_2 + OS_1 + OS_2 + OS_3 + OS_4 + 1S_5 + OS_6 = 500$$

$$x_1, x_2 \geq 0$$

Since we are minimizing cost, the objective function is multiplied by (-1) to change to maximization problem. thus, we have:-

Min Z = -Max(-Z), therefore, it implies that:-

Max Z = $-251,520x_1 - 220880x_2 + OS_1 + OS_2 + OS_3 + OS_4 + OS_5 + OS_6$

s.t

$$600x_1 + 200x_2 + 1S_1 + OS_2 + OS_3 + OS_4 + OS_5 + OS_6 = 1000$$

$$200x_1 + 110x_2 + OS_1 + 1S_2 + OS_3 + OS_4 + OS_5 + OS_6 = \frac{1000}{3}$$

$$5x_1 + 5x_2 + OS_1 + OS_2 + OS_3 + OS_4 + OS_5 + 1S_6 = 10$$

$$95x_1 + 95x_2 + OS_1 + OS_2 + 1S_3 + OS_4 + OS_5 + OS_6 = 250$$

$$100x_1 + 190x_2 + OS_1 + OS_2 + OS_3 + 1S_4 + OS_5 + OS_6 = 300$$

$$0x_1 + 400x_2 + OS_1 + OS_2 + OS_3 + OS_4 + 1S_5 + OS_6 = 500$$

$$x_1, x_2 \geq 0$$

Table 5: The first simplex tableau

BV	C_B	x_1	x_2	S_1	S_2	S_3	S_4	S_5	S_6	X_B
X_1	S_1 0	600	200	1	0	0	0	0	0	100
X_2	S_2 0	200	110	0	1	0	0	0	0	$\frac{1000}{3}$
X_3	S_3 -m	5	5	0	0	0	0	0	1	10
X_4	S_4 0	95	95	0	0	1	0	0	0	250
	S_5 0	100	190	0	0	0	1	0	0	300
X_5	S_6 0	0	400	0	0	0	0	1	0	500
X_6	Zj Cj	-5m -251520	-5m -220880	0 0	0 0	0 0	0 0	0 0	-m -m	
	Zj – Cj	251520 -5m	2200880 -5m	0	0	0	0	0	0	0

Table 6: The solution is optimal and feasible at the 4th iteration as shown in the table

BV	C_B	x_1	x_2	S_1	S_2	S_3	S_4	S_5	X_B

X_1	S_1 0	0	0	1	0	0	$\frac{40}{9}$	0	$\frac{2200}{9}$
X_2	S_2 0	0	0	0	0	0	1	0	$\frac{1000}{3}$
X_3	S_3 0	0	0	0	0	0	$\frac{-40}{9}$	1	$\frac{500}{9}$
X_4	S_4 0	0	0	0	0	1	0	0	60
	x_1, -251250	1	0	0	0	0	$\frac{-1}{9}$	0	$\frac{8}{9}$
X_5	x_2, -220880	0	1	0	0	0	$\frac{1}{90}$	0	$\frac{10}{9}$
X_6	-251250	-220880	0	0	0	0	$\frac{1532}{45}$	0	Z=468,995.60
	-251250	-251520	0	0	0	0	0	0	
	Zj – Cj	0	0	0	0	0	$\frac{1532}{45}$	0	

Table 7: The result of the objective function for each iteration is as follows:-

ITERATION	OBJECTIVE FUNCTION (VALUE) (₦)
0	0
1	3,000,000.00
2	1,401,100:00
3	520,800:00
4	468,995:60

Hence the cost is minimized when the objective function is N468, 995.60 with $x_1 = \frac{8}{9}, x_2 = \frac{10}{9}$

8. RESULT AND DISCUSSION

The theory and methodology of the minimization of cost of poultry feeds is used to formulate a model. The data was also used for analytical analysis. The analytical solution of the data showed that, the optimum solution was obtained after the 4th iteration, and was found to be ₦468,995.60 and the corresponding values of x_1 and x_2 were respectively $\frac{8}{9}$ & $\frac{10}{9}$. However, when the model was solved using the software (TORA optimization system version 2000-2007), the optimum solution was found to be the same as that of the analytical solution after just one 1-iteration, as ₦468, 995.6, when $x_1 = \frac{8}{9}$ and $x_2 = \frac{10}{9}$.

We therefore discussed as such:-

Small scale agriculture should be encouraged like poultry production reduced unemployment for our teeming youths and to increase the production of source of proteins like eggs and chickens. For an efficient production of poultry, determination of the cost of feeds using the linear programming approach is important.

9. CONCLUSION AND RECOMMENDATION

9.1 CONCLUSION

Even though the poultry farming business has been in existence for the past years, farmers have been suffering from high cost of the feeds. The poultry farmers can now derive the benefit of the linear programming technique, through cost minimization. However, research should also be carried out to check the high cost of rearing animals like goats, sheep, camel fish, so that the farmers can derive maximum profit and Nigerians can get these commodities at cheaper rates.

9.2 RECOMMENDATION

Sequel to the outcome of this research we hereby recommend the following to the authorities concerned.

1. Poultry and other related farmers should relate with research institutions to find out the best combinations to either minimize cost or to maximize profit.
2. Research institutions should be established closer to our farmers to facilitate growth of our economy.
3. Farmers should be encouraged more so as to engage in the production of more food, which is very rich in protein content.
4. Researchers should work on the areas like cost minimization of some animals like goats, sheep and camels etc.

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