APPLICATION OF MODEL ORDER REDUCTION TECHNIQUES IN ENHANCING ROTOR ANGLE STABILITY OF A POWER SYSTEM

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ABSTRACT
Modelling of complex dynamic systems, such as power system is usually muchcomplicated because of the complexity of the Rotor angle system.Therefore, approximationprocedures are often used to reduce such complexities, in order to come up with simplermodels than the original models, for easy control implementations. This becomes necessary as the control laws may be too complex, with regards to practical implementations. In this Paper, optimal Henkel-norm approximation was applied, and the complexity of the linear rotor angle system was reduced.As the level of reliability of any system depends upon the cost considerations and for the use to which the system will be put, manufacturers producing equipment where reliability is of paramount importance, will need to expend much time and effort in design, building, testing and inspection stages, ensuring that the reliability meets the requirements. This could notbe possible to achieve when the system's complexity is not reduced.

INTRODUCTION

Model order reduction is a branch of control theory that concerns with the reduction of complexity of dynamics systems by examining their properties while maintaining their input-output behaviour. The main goals of model order reduction are summarised as follows:

A. To simplify the best available model in light of the purpose for which the model is to be used, for instance, to design a control system that meets the requiredspecifications.
B. To hasten the simulation process during the design validation stage by using asmaller size model with most of the important system dynamics preserved.
C. Reduction of the control law complexity with little change in the controlperformance.

Description of large-scale systemsby mathematical models involves a set of first-order differential or difference equations. These models can be used to simulate the system response and predict behaviour. Sometimes, these mathematical models are also used to modify or control the system behaviour to conform to certain desired performance. In practical control engineering applications with the increase in the need for improved accuracy.

Mathematical models lead to high order and complexity. Although the well established modern control concepts are valid for any system order, they may not give fruitful control algorithms in control design. Moreover, working withvery high order modelinvolves computational complexity and need for high storage capability. Sometimes, the presence of small-time constants, masses, etc. may give rise to interaction among slow and fast dynamic phenomena with attendant ill-conditioning of stiff numerical problems. When analyzing and controlling these large-scale dynamic systems, it is extremely important to look for and to rely upon efficient, simplified reduced-order models which capture the main features of the full order complex model.

REVIEW OF MODEL ORDER REDUCTION TECHNIQUES

Let consider a large-scale dynamical system described by the linear time-invariant model

$\dot{x} = Ax + Bu$……………………………….(1)

$y = Mx$……………………………………...(2)

Where$x, uandy$ denote respectively the n dimensional state, $m$dimensionalinput and p dimensionaloutput vectors and $A, BandM$ the system, input and output matrices. For large-scale systems, the order $n$is quite large, and the intent of model order reduction is to obtain a simplified lower-order model whichpreserves the input and output behavior of the system. The reduced model of order $n_1 < n$ has same response characteristics as that of the original model with far fewer storage requirements and much lowerevaluation time.

The resulting model is given by,

$\dot{x}_r = A_r x_r - B_r u_r$……………………………(3)

$y_r - M_r x_r$……………………………………..(4)

Might be used to replace the original description in simulation studies or it might be used to design a reduced-order controller or observer. The application of Davison's order reduction technique, Marshall's model order reduction technique and Singular Perturbation technique has been explored. These techniques are briefly described in the following.

DAVISON'S TECHNIQUE

A structured approach to model order reduction was described in (Anand,1984) which approximates the original order n of the system to n_1 by neglecting the eigenvalues of the original system that are farthest from the origin and retains only the dominant eigenvalues and hence the dominant time constants of the original system are present in the reduced-order model. Initially, the system states are rearranged in such a manner that the eigenvectors corresponding to the states to be retained from (1) are placed first.

Let the state vector x is partitioned into dominant and non-dominant parts as x_1 which are considered to be retained and $\square_2$ which are to be ignored. Therefore the partitioned form of (1), (2) is

$$\begin{bmatrix}\dot{\square}_1\\ \dot{\square}_2\end{bmatrix} = \begin{bmatrix}\square_{11} & \square_{12}\\ \square_{21} & \square_{22}\end{bmatrix}\begin{bmatrix}\square_1\\ \square_2\end{bmatrix} + \begin{bmatrix}\square_1\\ \square_2\end{bmatrix}\square \ldots\ldots (5)$$

$$\square = [\square_1 \quad \square_2]\begin{bmatrix}\square_1\\ \square_2\end{bmatrix} \ldots\ldots (6)$$

Where the order of $\square_1 \square\square\square_1$ and the order of $\square_2\square\square\square - \square_1$. Further consider the representation of the system (5) ,(6) by the equivalent diagonal form,

$$\begin{bmatrix}\dot{\square}_1\\ \dot{\square}_2\end{bmatrix} = \begin{bmatrix}\breve{\square}_1 & 0\\ 0 & \breve{\square}_2\end{bmatrix}\begin{bmatrix}\square_1\\ \square_2\end{bmatrix} + \begin{bmatrix}\breve{\square}_1\\ \breve{\square}_2\end{bmatrix}\square \ldots\ldots (6)$$

$$\square = [\breve{\square}_1 \quad \breve{\square}_2]\begin{bmatrix}\square_1\\ \square_2\end{bmatrix}, \ldots\ldots (7)$$

Where order of $\square_1\square\square\square_1\square\square\square\square h\square\square\square\square\square_2\square\square\square - \square_1$

$$\breve{\square}_1 = \square\square\square\square.\begin{bmatrix}\square_1 & \square_2 \ldots & \square_{\square 1}\\ \square_{\square+1} & \square_{\square+2} \ldots & \square_{\square}\end{bmatrix} \ldots\ldots (8)$$

And the eigenvalues $\square_{\square}, \square = 1, 2, \ldots\ldots \square_1$ are to be retained in the approximate model. Let

$$\square = \square_{\square} = \begin{bmatrix}\square_{11} & \square_{12}\\ \square_{21} & \square_{22}\end{bmatrix}\begin{bmatrix}\square_1\\ \square_2\end{bmatrix} \ldots\ldots (9)$$

be the required linear transformation for obtaining the diagonal form representation. According to Davidson's method,(Anand,1984) the mode in $\square_2$ are non-dominant and therefore can be ignored, thus setting $\square_2 = 0$ in (9) gives a reduced-order model (3), (4) where

$\square_{\square} = \square_{11}\breve{\square}_1\square_{11}^{-1}$……………………………………………..(10)

$\square_{\square} = \square_{11}\breve{\square}_1$………………………………………………….(11)

$\square_{\square} = \breve{\square}_{11}\square_1^{-1}$…………………………………………………(12)

and $\square_2 = \square_{21}\square_{11}^{-1}\square_1$……………………………………….(13)

Thus the original nth-order model is approximated by $\square_1$□horder model. The first $\square_1$ state variables of the original model are approximated by the state variables of the reduced-order model and $\square - \square_1$ state variables are expressed in terms of first $\square_1$ states variables by (13).

MARSHALL'S TECHNIQUE

An alternate method for the computation of the reduced-order model is proposed in (Kokotovic, 1999) in which it assumed that $\dot{\square}_2 = 0$ in (6) which then yields

$\dot{\square}_1 = \breve{\square}_1\square_1 + \breve{\square}_1\square$……………………………………………(14)

$0 = \breve{\square}_2\square_2 + \breve{\square}_2\square$……………………………………………..(15)

From (9), we have $\square = \square^{-1}\square = \begin{bmatrix}\square_{11} & \square_{12}\\ \square_{21} & \square_{22}\end{bmatrix}\begin{bmatrix}\square_1\\ \square_2\end{bmatrix}$……………(16)

Then

From (15), we obtain $\square_2 = \square_{22}^{-1}\square_{21}\square_1 = \square_{22}^{-1}\breve{\square}_2^{-1}\breve{\square}_2\square$………(17)

Substituting the solution of $\square_2$ from (17) into (5), the reduced-order model is obtained as (3) and (4), where

$\square_{\square} = \square_{11} - \square_{12}\square_{22}^{-1}\square_{21}$……………………………………(18)

$\square_{\square} = \square_{11} - \square_{12}\square_{22}^{-1}\breve{\square}_2^{-1}\breve{\square}_2$………………………………….(19)

And $\square_2 = -\breve{\square}_{22}^{-1}(\square_{21}\square_1 - \breve{\square}_2^{-1}\breve{\square}_1\square)$…………………………..(20)

Again the original $\square^{\square h}$ order model is approximated by $\square_1^{\square h}$ order model. The first $\square_1$ state variables of the original model are approximated by the state variables of the reduced-order model and the $\square - \square_1$state variables are expressed in terms of the first $\square_1$ state variables by (20)

SINGULAR PERTURBATION TECHNIQUES

In Linear time-invariant models of large scale systems, the interaction of slow and fast modes is a common feature, and it leads the mathematical models to be ill-conditioned in control design. Singular Perturbation analysisprovides a simple means to obtain approximate solutions to the original system as well as it alleviates the high dimensionality problem (Bikash, 2005). In this method, both the slow and fast modes are retained, but analysis and design problems are solved in two stages. By a suitable regrouping of the state variables, the original higher-order system can be expressed into standard singularly perturbed form in which the derivatives of some of the states are multiplied by a small positive scalar $\square$, i.e.,

$\dot{\square}_{\square} = \square_{11}\square_{\square} + \square_{12}\square_{\square} + \square_1\square, \square_{\square}(0) = \square_{10}$,……………………………...(21)

$\in \dot{\square}_{\square} = \square_{21}\square_{2} + \square_{22}\square_{\square} + \square_{2}\square, \square_{\square}(0) = \square_{20}$……………………………(22)
and $\square = \dot{\square}_{1}\square_{\square} + \dot{\square}_{2}\square_{\square}$…………………………………………………(23)
where $\square_{\square} \in \square^{\square_1}$ is the slow state vector , $\square_{\square} \in \square^{\square_2}$ is the fast state vector.
Let $\square(\square) = \{\square_1, \square_2 \ldots \ldots . \square_{\square}\}$……………………………………………(24)
by setting the parasitic parameter $\square = 0$ in (22), it yields
$0 = \square_{21}\breve{\square}_{\square} + \square_{22}\breve{\square}_{\square} + \square_{2}\square$………………………………………………(25)
Where $\breve{\square}_{\square}, \breve{\square}_{\square}$ are the variables of the system (21), (22) when $\square = 0$. If $\square_{22}^{-1}$ exists, then the solution of $\breve{\square}_{\square}$ into (21) results in the reduced-order model of order $\square_1$ as
$\dot{\square}_{\square} = \square_{0}\square_{\square} + \square_{0}\square_{\square}$……………………………………………………(26)
$\square_{\square} = \square_{0}\square_{\square} + \square_{0}\square_{\square},$…………………………………………………(27)
Where
$\square_{\square} = \breve{\square}_{\square}$…………………………………………………………….(28)
$\square_{\square} = \breve{\square}$………………………………………………………………....(29)
$\square_{\square} = \square_{11} - \square_{12}\square_{22}^{-1}\square_{21}$…………………………………………………..(30)
$\square_{\square} = \square_{1} - \square_{12}\square_{22}^{-1}\square_{2}$……………………………………………………(31)
$\square_{\square} = \breve{\square}_{1} - \breve{\square}_{2}\square_{22}^{-1}\square_{21}$…………………………………………………..(32)
$\square_{\square} = -\breve{\square}_{2}\square_{22}^{-1}\square_{2}$…………………………………………… (34)
And a fast system with order $\square - \square_1$ given by
$\square\dot{\square}_{\square} = \square_{22}\square_{\square} + \square_{2}\square_{\square}$…………………………………………..(35)
$\square_{\square} = \breve{\square}_{2}\square_{\square}$………………………………………………………..(36)
Where
$\square_{\square} = \square_{\square} - \breve{\square}_{\square}$………………………………………………..(37)
$\square_{\square} = \square - \breve{\square},$……………………………………………………(38)
Therefore, eigenvalues of the original system are,
$\square(\square) = \square(\square_{\square}) \cup \square(\frac{\square_{22}}{\square})$……………………………(39)

BALANCED TRUNCATION

This is a reduction technique in which the truncated model is the same as the original model at infinite frequency. In this technique, the state which corresponds to fast modes is truncated or removed, and the poles of the removed model are a subset of the poles of the originalmodel, and this preserves any physical interpretation they might have in the new model (truncated model).

BALANCED RESIDUALIZATION

In this technique, the derivatives of all the states are set to zero, and this is achieved by residualized the state with least controllability and observability. By doing this, the steady state gain of the system is preserved. Let consider the state space equations below:

$$\dot{\square}_1 = \square_1\square_1 + \square_{12}\square_2 + \square_1\square \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . (40)$$
$$\dot{\square}_2 = \square_{21}\square_1 + \square_{22}\square_2 + \square_2\square \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (41)$$
$$\square = \square_1\square_1 + \square_2\square_2 + \square\square \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (42)$$

By setting $\dot{\square}_2 = 0$, $\square_2$can be solved in terms of $\square_1$ and $\square$

$$0 = \square_{21}\square_{1} + \square_{22}\square_{2} + \square_{2}\square \ldots\ldots\ldots (43)$$

Therefore,

$$\square_{2} = -\square_{22}^{-1}(\square_{21}\square_{1} + \square_{2}\square) \ldots\ldots\ldots (44)$$

Substituting into $\square_1$ dynamic, yields:

$$\dot{\square}_{1} = (\square_{11} - \square_{12}\square_{22}^{-1}\square_{21})\square_{1} + (\square_{1} - \square_{12}\square_{22}^{-1}\square_{2})\square \ldots (45)$$

$$\square = (\square_{1} - \square_{2}\square_{22}^{-1}\square_{21})\square_{1} + (\square - \square_{2}\square_{22}^{-1}\square_{2})\square \ldots\ldots (46)$$

If $\square_{22}$is assumed to be an invertible matrix and let

$$\square_{\square} = (\square_{11} - \square_{12}\square_{22}^{-1}\square_{21}) \ldots\ldots\ldots (47)$$

$$\square_{\square} = (\square_{1} - \square_{12}\square_{22}^{-1}\square_{2}) \ldots\ldots\ldots (48)$$

$$\square_{\square} = (\square_{1} - \square_{2}\square_{22}^{-1}\square_{21}) \ldots\ldots\ldots (49)$$

$$\square_{\square} = (\square - \square_{2}\square_{22}^{-1}\square_{2}) \ldots\ldots\ldots (50)$$

Then the reduced-order model,

$\square_{\square}(\square) = (\square_{\square}, \square_{\square}, \square_{\square}, \square_{\square})$ is called a balanced residualization of the initial system

$$\square(\square) = (\square, \square, \square, \square).$$

OPTIMAL HANKEL NORM APPROXIMATION

The Hankel norm of any stable transfer function $\square(\square)$can be derived when an input $\square(\square)$ is applied, and the output $\square(\square)$for $\square > 0$is measured. A $\square(\square)$is selected so that the ratio of the two norms of the two signals is maximized.

$$\|\square(\square)\|_{\square} = \square\square\square_{\square(\square)}\left(\sqrt{\int_{0}^{\infty} \|\square(\square)\|_{2}^{2}\square\square}\right) / \left(\sqrt{\int_{-\infty}^{0} \|\square(\square)\|_{2}^{2}\square\square}\right) \ldots\ldots\ldots (51)$$

Given that the stable system $\square(\square)$is of order n, the reduced model order $\square_{h}^{\square}(\square)$of degree $\square$can be found such that the Hankel norm of the approximation error $\|\square(\square) - \square_{h}^{\square}(\square)\|_{h}$is minimized. The Hankel norm of $\square(\square)$is defined as:

$$\|\square(\square)\|_{\square} = \sqrt{\square(\square\square)} \ldots\ldots\ldots (52)$$

Where $\square$is the spectral radius (absolute value of maximum eigenvalue), P and Q are the Controllability and observability of gramians of$\square(\square)$. Therefore in the optimization, we look for an error which is near to being completely unobservable and completelyuncontrollable.

IMPLEMENTATIONS

In this paper, the original system is nonlinear. Using Matlab, the linearized model is obtained with 12 outputs, 4 inputs and 55 states which is then loaded into the Matlab for reduction Implementation. But before attempting model order reduction, it exists to inspect pole and zero locationsof the model.

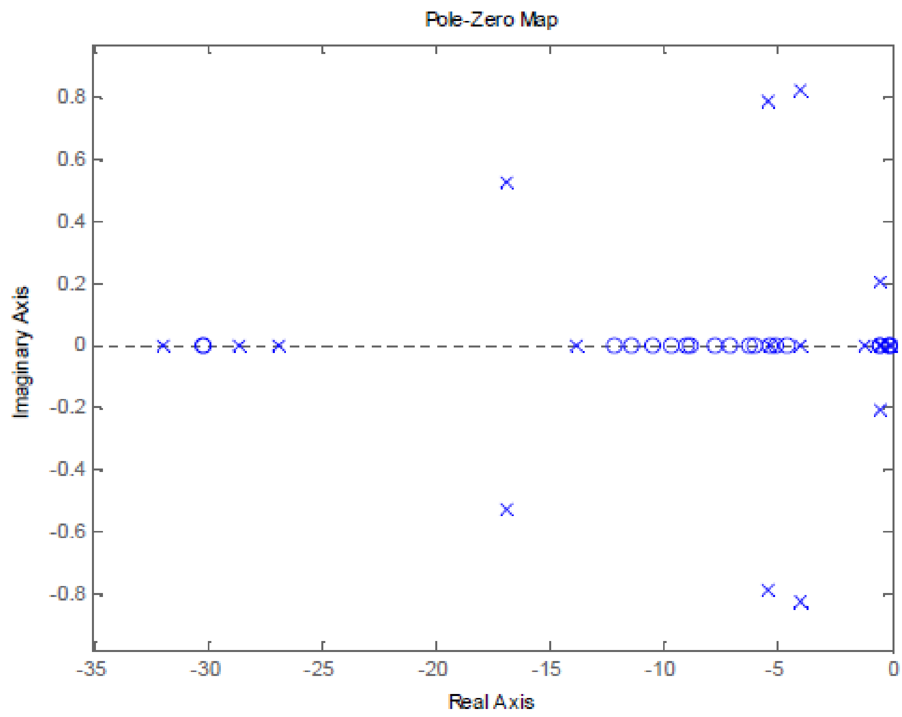

Figure 1: pole-zero map of the original model

Inspecting the pole and zero locations, it can be observed that the model displays nearpole-zero cancellations. For this reason, it can be considered as a good candidate for model reduction. In control theory, eigenvalues define system stability, whereas Hankel singular valuesdefine the 'energy' of each state in the system. Keeping larger energy states of a systemmaintains most of its characteristics in terms of stability, frequency, and timely responses.To find a low-order reduction for the model, an appropriate order for the reduced modelshould be selected by examining the relative amount of energy per state using Hankel Singular value (HSVD) plot.

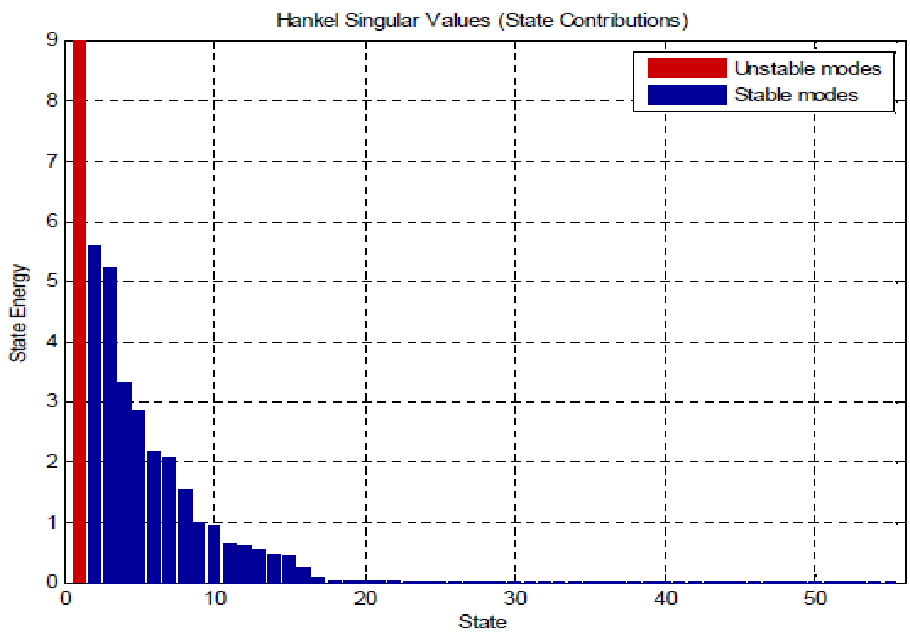

Figure 2: HSV Plot for the Model

The figure shows large and small Hankel singular values associated with the state'scontribution to the input/output behaviour of the system. It indicates that those states withsmall Hankel singular values contribute little to the input/output behaviour of the system. Hence, discarding the states corresponding to the smallest Hankel singular values shouldhave little impact on the error in the resulting reduced-order model.The last 33 states can be discarded, and this results in the 22nd order model.In order to facilitate the visual inspection of the approximation error, a singular value plotis used.

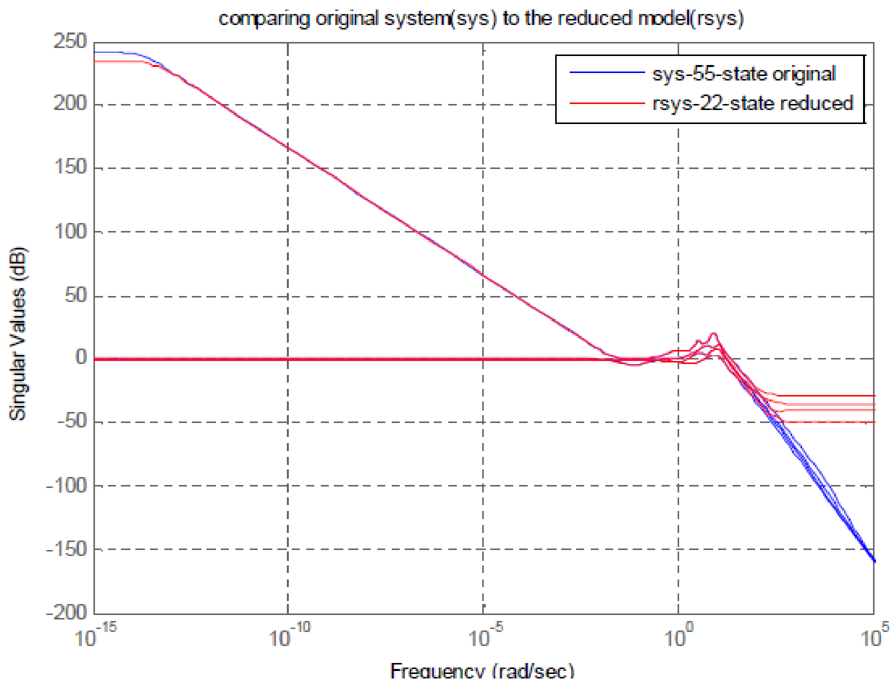

Figure 3: A Sigma Plot Comparing the original Model to the reduction error

Looking at the sigma plot response, it can be clearly seen that at low frequency, thereduced model overlapped the original model up to a frequency of about 200rad/sec(31.83Hz) where it deviated. The reduction error can, therefore, be seen as minimal andhence, the reduced-order model can be considered to be a good approximation of theoriginal model.Let now try to reduce the model order further by discarding more states in order to seehow to further order reduction has to impact on the reduction error, so as to justify the choice of22nd order model. The model is reduced to 10th, 15th and 22ndorders.

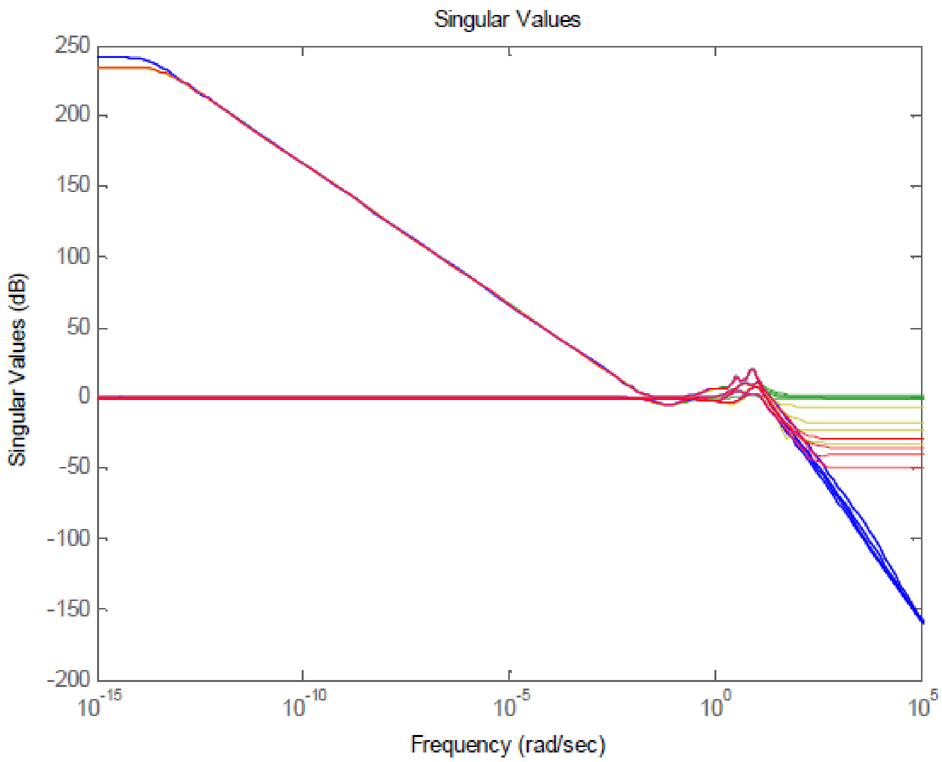

Figure 4: Sigma Plot of original Model and 10th, 15th, 22nd reduced Model

From the plot response, it can be observed that the approximation error increases as themodel order decreases. This shows that the 22ndorder model can be considered as it lookspromising compared to 10th and 15th orders with respect to minimization in the structured error||□□□ − □□□□||which also ensures the stability of the reduced model.The further test had been carried out in order to make sure that the reduced-order modelselected is of appropriate approximation of the original model. This was done bysimulation and comparing the original model with the reduced models (10th, 15th, and 22ndorders).

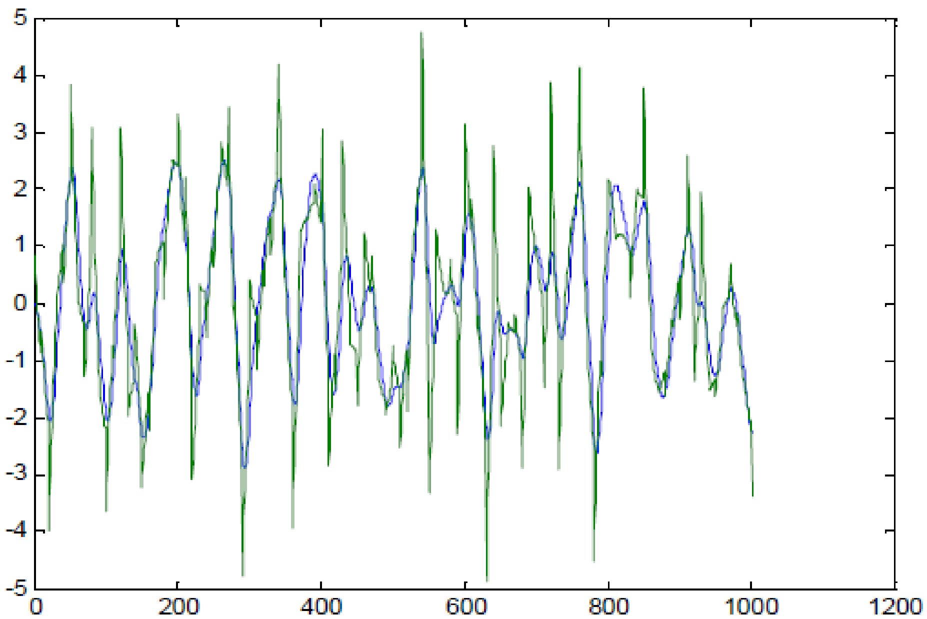

Figure 5: Comparison of the original model with the reduced 10thorder model

In the above figure, the response of the original model is indicated in blue while that of the reduced model is represented by green. It was observed that the two models do notmatch appropriately and therefore, there is more need to minimize the structurederror‖□□□ − □□□□‖.

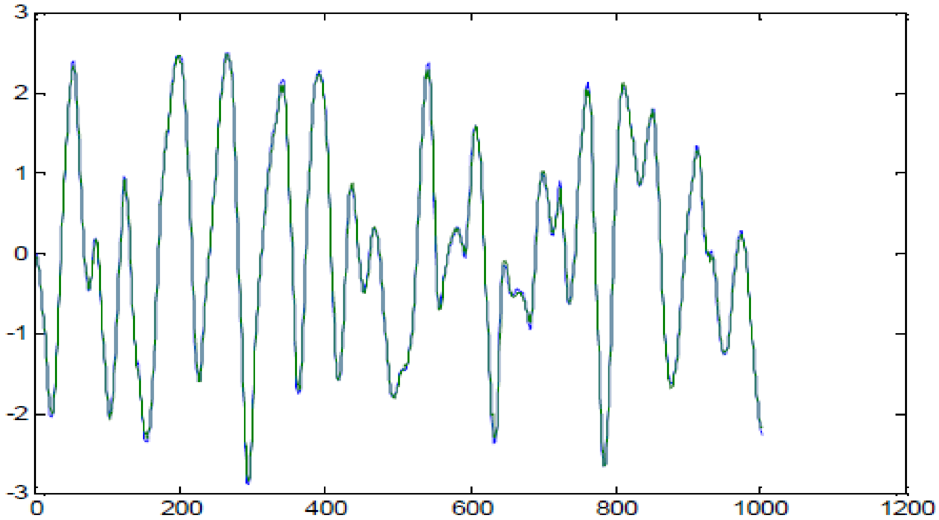

Figure 6: Comparison of the original model with the reduced 15thorder model

In the above figure, with the original model response indicated in blue during the response in Green representing the reduced model 15thorder, the reduction error has been minimized further compared to 10thorder, although some mismatches are still observed.

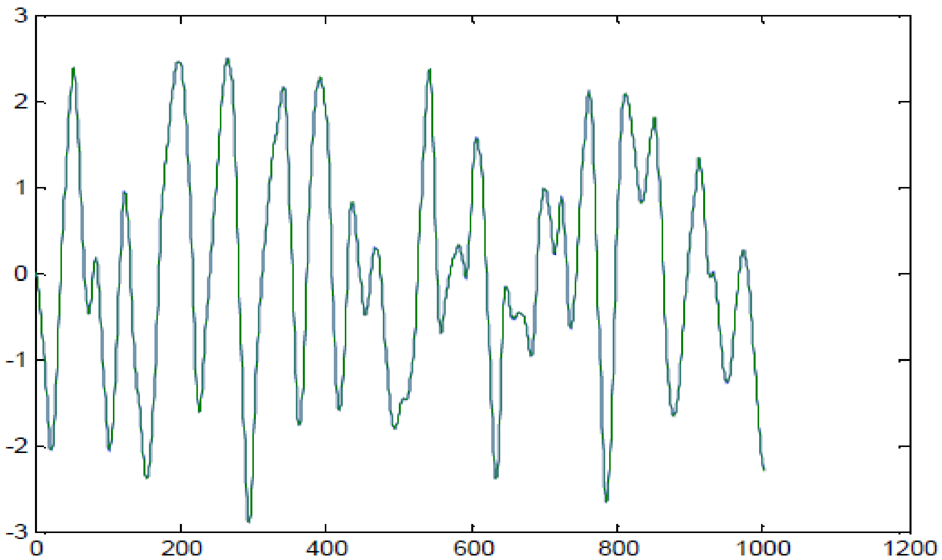

Figure 7: Comparison of the original model with the reduced 22ndorder model

In the above figure, it can be seen that the 22ndorder reduced model nearly matched completely the original model, and this shows that the reduction error is at minimal. The 22ndorder reduced model can, therefore, be considered as the best approximation of the Original model (55thorder).

COMPARISON OF THE GENERATORS SPEED RESPONSE PRODUCED BY THE FULL-ORDER LINEAR MODEL AND REDUCED ORDER MODEL

An open-loop experiment had been conducted with the inputs 4 random signals (normally distributed) having different seed numbers and sampling time set to 0.1 seconds.The four measured variables of the speed produced from the full-order linear model of thepower system are then compared to those produced by the reduced-order model.

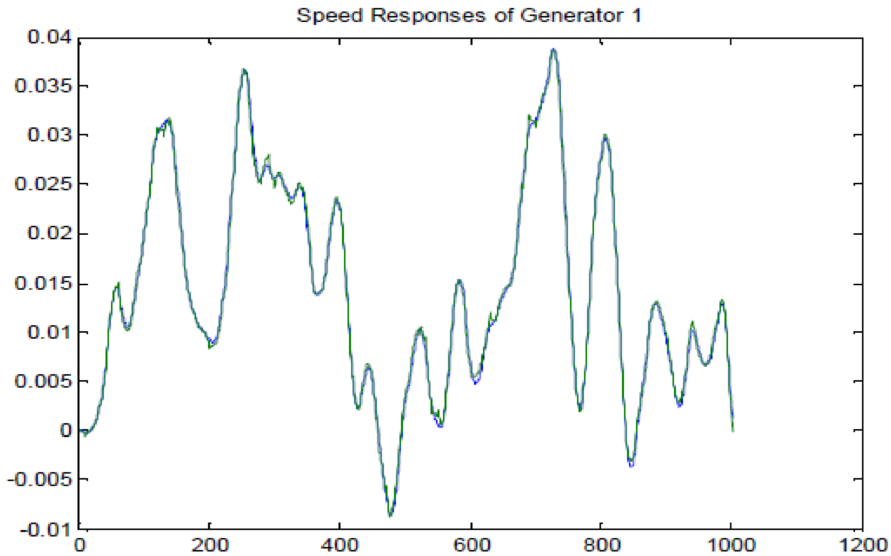

Figure 8: Speed response of Generator 1

The speed response of Generator 1 produced from the full-order linear model is indicated by the blue response, while that of the same Generator but produced by the reduced-order model is indicated by the green response. The two-speed responses closely matched.

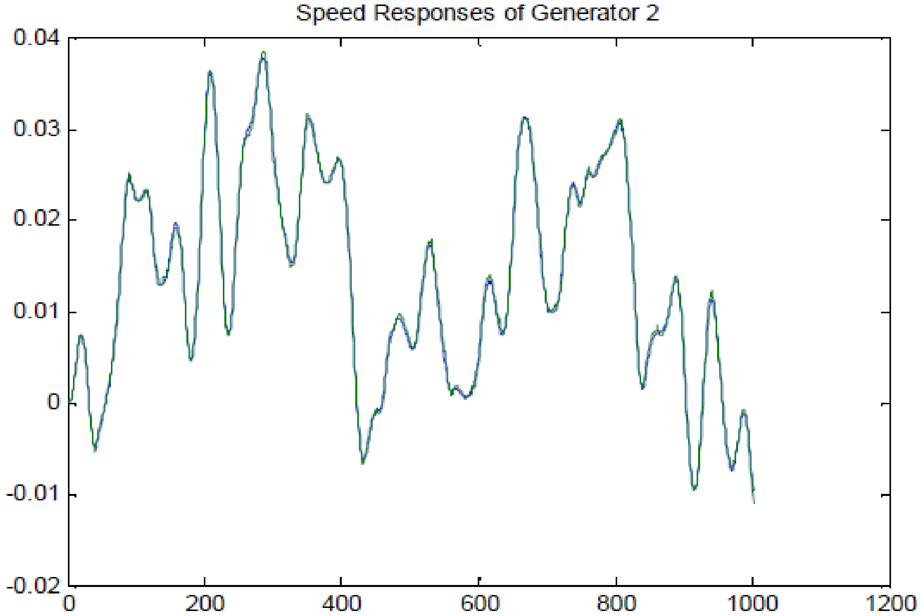

Figure 9: Speed response of Generator 2

In the above plot response of Generator 2, it has also been shown that the two speed Variables although produced from different linear model order closely matched with Minimal approximation error.

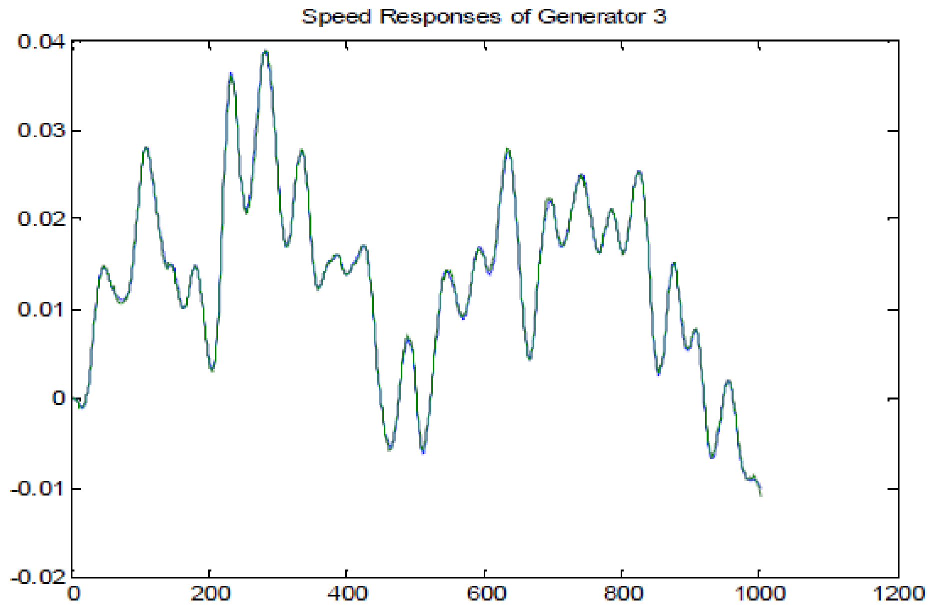

Figure 10: Speed response of Generator 3

The variables responses of the speed for Generator 3 also show close matching for the two Cases.

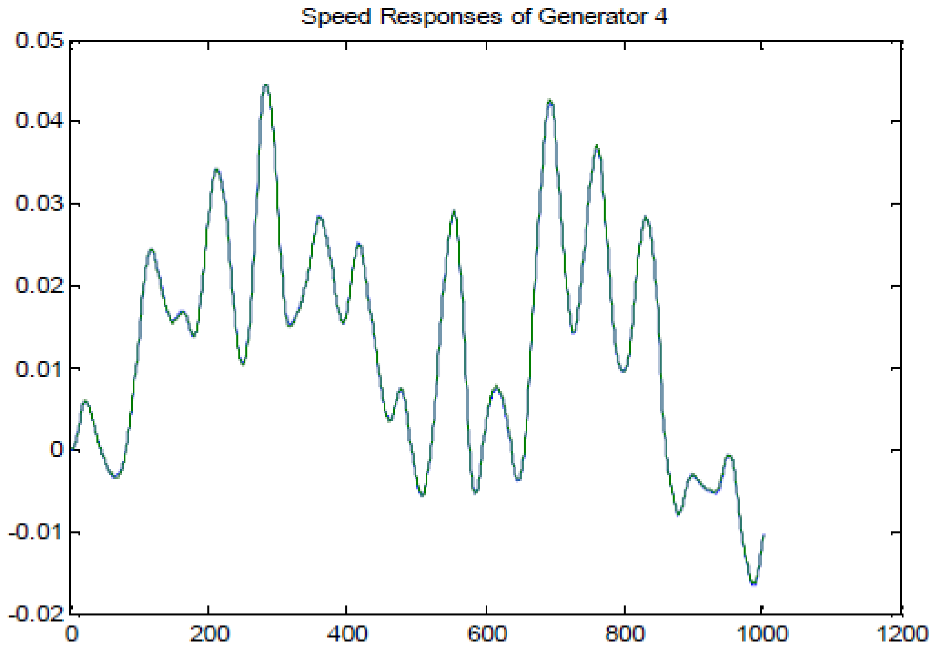

Figure 11: Speed response of Generator 4

The responses for the two cases in Generator 4 also indicated a close matching withminimal approximation error.

CONCLUSION

In either of the Generator, it has been shown that, the speedy response of the full-order (55th States) linear model closely matched that of the reduced-order (22nd states) model. This alsoindicated that the original linear model (55 states) of the power system could be accuratelyrepresented by a reduced model (22 states) with all the input/output behaviour of the original linear model preserved.

Due to demand from military, communication, and aerospace sectors, for example, a growing need to develop complex equipment for high reliability has been placed. Some equipment or systems are complex in nature, due to either their multivarious features or the multi-varied functions they are expected to perform. There is a need for equipment/systems to have very high reliability, and this may not be possible to achievewithout reducing the system's complexity prior to design.

From the system's manufacturer point of view, this research will tremendously assist when considering redundancy method, which is the provision of more than one means of getting an equipment/system to perform a given function.In the context of reliability, it represents a means of enhancing system reliability. It also prolongs the operating time of the equipment that cannot be maintained like spacecraft significantly. However, its application has limitations such as weight, space, complexity, cost and time to design and maintenance cost.All these would have been possible when the system's complexity is reduced.

REFERENCE

Anand, D.K. (1984).*Introduction to control systems* (2nd Edition), Pergamon press ltd., Headington Hill Hall, Oxford OX3, OBW, England.

P. Kokotovic, Khali, H.K, and O'reilly J. (1999). *Singular Perturbation Methods in Control: analysisand design*. vol. 25, siam.

Belanger, Pierre R. (1985). *Control engineering, a modern approach*. Sanders college publishing, Harcourt Brace and company.

Bikash, P., Balarko Chaudhuri. (2005). *Robust Control in Power Systems*. Springer, pp1, pp5-9.

A Rajasekhar, A.P. Tiwari, Madhu. N. Belur. (May 2015). *Application of Model order reduction Techniques to Space-Time kinetics Model of AHWR*. 2015 International Conference on Industrial Instrumentation and Control (ICIC) College of Engineering Pune, India.

Eydgahi, A. J. Habibi and Moshiri B.(2003). *A Matlab Toolbox for teaching model order reduction techniques*. International conference on engineering education, Valencia, Spain.

Glover, K. (1984). *A tutorial on Hankel norm approximation*. Illinois.edu.

Marshall, S. (1996). *An approximate method for reducing the order of a linear system*. Control Engineering, Vol. 10, pp. 639-648.

Glover, K. (1984). *Optimal Hankel-norm approximations of linear multivariable systems and their* $-\infty$ *bound*. Int. J. Control, 39(6): 115-193.

Hung, Y., Glover, K. (1986). *Optimal Hankel Norm Approximation with first-order stable weighting functions.* North-Holland, system and control letters 7.

Kundur, P. (1994). *Power System Stability and Control.* Mc Graw-Hill, Inc.

Mchowski, J., J.W. Bialek and J.R. Bumby. (1997). *Power System Dynamics: Stability and Control.* (Second edition), John Wiley & Sons.

Seborg, D.E., Thomas F. Edgar, Duncan A. Mellichamp. (1989). *Process Dynamics and Control*. John Wiley & Sons.

Skogestad, S. and Ian Postlethwaite. (1987). *Multivariable Feedback Control*: Analysis and Design, (Second edition), John Wiley & Sons.

E.J. Davidson. (1996). *A method for simplifying linear dynamic systems.* IEEE Tran. Auto control, vol. 11, no. 1. Pp. 96-100.