TRANSIENT FREE- CONVECTION FLOW OF EXOTHERMIC FLUID WITH MAGNETIC FIELD OVER A HEATED/COOLING

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ABSTRACT
This paper theoretically consider steady natural convection flow of exothermic fluid with magnetic field over a asymmetrically. The flow is assumed to be laminar, viscous, incompressible and in an electrically conducting fluid. The flow is governed by momentum and energy equations. The governing steady equations are solved analytically using perturbation series method. The solutions are obtained for different values of Magnetic field (M), Porosity parameter (K), Reactant consumption parameter (λ), Activation energy parameter (ε) and Buoyancy distribution parameter (r). The result of temperature, velocity, Nusselt number and skin friction are obtained and discussed using line graphs.It was observed that an increase in the values of reactant consumption parameter and porosity parameter increase velocity profiles, while an increase in magnetic parameter decreases. An increase in porosity parameter and buoyancy distribution parameter, also increase heat transfer.

Keyword: Free convention, Transient, viscous reactive fluid and magnetic field

INTRODUCTION

The effect of a uniform transverse magnetic field on the free-convective flow of a viscous, incompressible and electrically conducting fluid past on infinite non-conducting, vertical porous plate when the fluid is subjected to a suction of uniform velocity was carried out by Nanousis *et al.* (1980). The flow of an electrically conducting fluid through a channel or circular pipe in the presence of a transverse magnetic field is encountered in a variety of applications such as magneto hydrodynamic (MHD) generators, pumps, accelerations and flow meters, MHD flow of a viscous, Newtonian, incompressible, electrically-conducting fluid through an isotropic, homogenous porous medium located in the annular zone between two concentric rotating cylinders in the presence of a radial magnetic field and so on.

Chandran *et al.* (2005) studied natural convection near a vertical plate with ramped temperature and obtained two different solutions one valid for the fluid of Prandtl numbers different from unity and the other for which the Prandtl number is unity. They obtained analytical solutions under the assumption that the velocity and temperature conditions on the wall are continuous and wall defined. They concluded that the solutions for the non-dimensional velocity and temperature variables depend upon the Prandtl number of the fluid and the expression of the fluid velocity is not uniformly valid for all values of Pr. Bianco *et al.* (2006) investigated natural convection in a vertical, convergent and symmetrically heated channel by taking in to consideration radiation effect. In another article, Langelloto *et al.* (2007) presented numerical investigation of natural convection in a convergent vertical channel in order to study the thermal and fluid dynamic behavior of the transient regime in this configuration. Jha and Ajibade (2010) investigated the transient natural convection flow between vertical parallel plates having iso-thermal and adiabatic conditions on the vertical plates. Jha *et al.* (2012) studied a theoretical analysis of natural convection flow of heat generating/ absorbing fluid near a vertical plate with ramped temperature.

Numerical and analytical solution of the developing laminar free convection of a micropolar fluid in a vertical parallel plate channel with asymmetric heating are presented Chamkha *et al.* (2002). The analyzed combined effects of magnetic field and viscous dissipation on convective heat and mass transfer flow through a porous medium in a vertical channel in the presence of heat generating sources by Balasubramanyam *et al.* (2010). Chamkha (2003) examined the boundary layer of a MHD flow when the heat generation is linear in temperature. Asymmetric temperature and convection boundary conditions are applied to the walls of the channel. Solutions of the coupled non- linear governing equations are obtained for different values of the buoyancy ratio and various material parameter of the micropolar fluid and magnetic parameters. Uwanta and Hamza (2014) investigated unsteady natural convection flow of reactive, viscous, electrically conducting fluid past impulsively moving vertical plates in the presence of uniform transverse magnetic field. Makinde and Anwar (2010) reported the inherent irreversibility and thermal stability in a reactive electrically conducting fluid following steadily through a channel with isothermal walls in another article. Kumar *et al.* (2013) investigated MHD double diffusive and chemically reactive flow in a porous medium.

MATHEMATICAL FORMULATION

We consider the motion of a steady two dimensional viscous,incompressible and electrically conducting fluid past a finite non-conducting vertical porous flat plate. A magnetic field of uniform strength is assumed to be applied in the direction perpedicular to the direction of flow. Reynold number is very small with corresponding to negligible induced magnetic field compared to the externally applied.

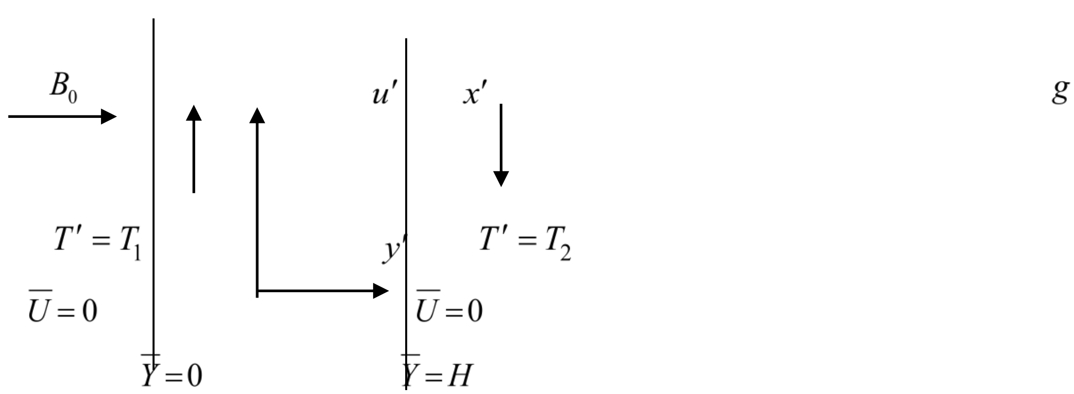

Figure 1 Schematic of the problem

The flow is assumed to be ϰ'-dire ction which is taken along the vertical plate in the upward direction. The y'-axis is taken to be normal to the plate. A time t' ≤0, the fluid and plate are assumed to be at rest and the fluid and plate temperature are T'_∞. The governning equations for the flow under these assumption are shown in figure 1.

$$v\frac{d^2\overline{U}}{d\overline{Y}^2}+g\beta\left(T'-T_0\right)-\frac{\sigma B_0^2H^2\overline{U}}{H^2\rho}-\frac{v\overline{U}}{k}=0 \quad (1)$$

$$\frac{d^2T'}{d\overline{Y}^2}+QC_oA\exp\left(\frac{-E}{RT'}\right)=0 \quad (2)$$

Subject to the boundary conditions

$$\left.\begin{array}{l}\overline{U}=0,\ T'=T_1 \quad at\ \overline{Y}=0\\ \overline{U}=0,\ T'=T_2 \quad at\ \overline{Y}=H\end{array}\right\} \quad (3)$$

Where, T_0 is the initial fluid and wall Temperature, T' is the dimensional temperature of fluid, Q is the heat of reaction, A is the rate constant, E is the activation energy, R is the universal gas constant, C_0 is the initial concentration of the reactant species, V is the kinematic viscosity, H is the gap between the plate. T_1 is the temperature of the hot wall and T_1 is the temperature of the cooled wall as shown in Figure (1).

The non-dimensional variables and parameter for the flow problem are

$$\left.\begin{array}{l}u=\dfrac{\overline{U}v\mathrm{E}}{g\beta H^2RT_o^2},\ y=\dfrac{\overline{Y}}{H},\ M=B_0h\sqrt{\dfrac{\sigma}{v\rho}},\varepsilon=\dfrac{RT_0}{E},\theta=\dfrac{\mathrm{E}}{RT_0^2}(T'-T_0),\\ \lambda=\dfrac{QC_0AEH^2}{RT_0^2}\exp\left(\dfrac{-E}{RT_0^2}\right),k=\dfrac{K}{H^2},r=\dfrac{\mathrm{E}}{RT_0^2}(T'-T_0),E=\dfrac{RT_0^2}{(T'-T_0)}\end{array}\right\} \quad (4)$$

Using eqn (4) in eqn (1) and eqn (2) the dimensioless momentum and energy equations are:

$$\frac{d^2u}{dy^2}-\left(\mathrm{M}^2+\frac{1}{K}\right)u+\theta\ =\ 0 \quad (5)$$

$$\frac{d^2\theta}{dy^2}+\lambda\ \exp\left(\frac{\theta}{1+\varepsilon\theta}\right)=0 \quad (6)$$

The boundary conditions in dimensionless form are

$$\left.\begin{array}{l}u=0,\ \theta=1 \quad at\ \ y=0\\ u=0,\ \theta=r \quad at\ \ y=1\end{array}\right\} \quad (7)$$

ANALYTICAL SOLUTIONS

We now reduce the governing equations of the present problem to a form that can be solved in a closed form. This is done by employing a regular perturbation method, taking the power series expansion in the reactant consumption parameter λ, as follows:

$$\theta(y)=\theta_0(y)+\lambda\theta_1(y)+\lambda^2\theta_2(y)+\lambda^3\theta_3(y) \tag{8}$$

$$u(y)=u_0(y)+\lambda u_1(y)+\lambda^2 u_2(y)+\lambda^3 u_3(y) \tag{9}$$

Substituting the solution series (8) and (9) into equations (5) and (6) and collecting the coefficients of like powers of λ the following set of differential equations is obtained

$$\left.\begin{aligned}&\theta_0''(y)=0, && \theta_1''(y)=-1\\ &\theta_2''(y)=-\theta_1 && \theta_3''(y)=-\theta_2+\theta_1^2\left(\varepsilon-\frac{1}{2}\right)\end{aligned}\right\}$$

$$\left.\begin{aligned}&u_0''(y)-\left(M^2+\frac{1}{K}\right)u_0(y)=-\theta_0(y),\ \ u_1''(y)-\left(M^2+\frac{1}{K}\right)u_1(y)=-\theta_1(y)\\ &u_2''(y)-\left(M^2+\frac{1}{K}\right)u_2(y)=-\theta_2(y), u_3''(y)-\left(M^2+\frac{1}{K}\right)u_3(y)=-\theta_3(y)\end{aligned}\right\}$$

(11)

With the corresponding boundary conditions reduced to

$$\left.\begin{aligned}&u_0(y)=u_1(y)=u_2(y)=u_3(y)=0,\\ &\theta_0(y)=\theta_1(y)=\theta_2(y)=\theta_3(y)=0\end{aligned}\right\} at\ y=0$$

$$\left.\begin{aligned}&u_0(y)=u_1(y)=u_2(y)=u_3(y)=0,\\ &\theta_0(y)=\theta_1(y)=\theta_2(y)=\theta_3(y)=0\end{aligned}\right\} at\ y=1$$

(12)

Solving equations (10) and (11) with the corresponding boundary conditions (12) the following solution of the velocity and temperature are obtained

$$\begin{aligned}\theta(y)=&(r-1)y+1+\lambda\left\{\frac{1}{2}\left(y-y^2\right)\right\}+\lambda^2\left\{-\frac{1}{24}\left(2y^3-y^4\right)+\frac{y}{24}\right\}\\ &+\lambda^3\left\{A_1 y-A_2\left(5y^3-3y^5+y^6\right)+A_3\left[5y^4-6y^5+2y^6\right]\right\}\end{aligned} \tag{13}$$

$$\begin{aligned}u(y)=&L_0e^{xy}-L_2e^{-xy}+f_0+f_1y+\lambda\left\{L_2e^{xy}-L_3e^{-xy}+f_2+f_3y+f_4y^2\right\}\\ &+\lambda^2\left\{L_4e^{xy}-C_5e^{-xy}+f_5+f_6y+f_7y^2+f_8y^3+f_9y^4\right\}\\ &+\lambda^3\left\{\begin{aligned}&L_6e^{xy}-L_7e^{-xy}+f_{10}+f_{11}y+f_{12}y^2+f_{13}y^3+f_{14}y^4\\ &+f_{15}y^5+f_{16}y^6\end{aligned}\right\}\end{aligned} \tag{14}$$

Rate of Heat Transfer and Skin Friction

The heat transfer which is represented by the Nusselt number Nu and the skin friction τ are similarly obtained by differentiating the Temperature and the Velocity with respect to y as follows.

$$Nu = \frac{d\theta}{dy}|_{y=0} = (r-1) + \frac{\lambda}{2} + \frac{\lambda^2}{24} + \lambda^3 A_1 \tag{15}$$

$$Nu = \frac{d\theta}{dy}|_{y=1} = (r-1) - \frac{\lambda}{2} - \frac{\lambda^2}{24} + \lambda^3 \left(A_1 - 6A_2 + 2A_3\right) \tag{16}$$

$$\begin{aligned}\tau_0 = \frac{du}{dy}|_{y=0} &= x\left(L_0 + L_1\right) + f_1 + \lambda\left\{x\left(L_2 + L_3\right) + f_3\right\} \\ &+ \lambda^2\left\{x\left(L_4 + L_5\right) + f_6\right\} + \lambda^3\left\{x\left(L_6 + L_7\right) + f_{11}\right\}\end{aligned} \tag{17}$$

$$\begin{aligned}\tau_1 = \frac{du}{dy}|_{y=1} &= x\left(L_0 e^{x} + L_1 e^{-x}\right) + f_1 + \lambda\left\{x\left(L_2 e^{x} + L_3 e^{-x}\right) + f_3 + 2f_4\right\} \\ &+ \lambda^2\left\{x\left(L_4 e^{x} + L_5 e^{-x}\right) + f_6 + 2f_7 + 3f_8 + 4f_9\right\} \\ &+ \lambda^3\begin{Bmatrix} x\left(L_6 e^{x} + L_7 e^{-x}\right) + f_{11} + 2f_{12} + 3f_{13} \\ +4f_{14} + 5f_{15} + 6f_{16}\end{Bmatrix}\end{aligned} \tag{18}$$

RESULTS AND DISCUSSION

The result of the flow model are obtained in terms of temperature, velocity, skin friction and rate of heat transfer, the selected graphs of the essential parameter and in value in the investigator are presented in figures. In order to set physical insight of the problem the above physical quantities are computed for different values of the governing parameter, the reactant consumption parameter (λ), magnetic field (M), porosity parameter (K), activation energy parameter (ε) and bouyancy distribution parameter (r). It may be noted that the key parameter r arises in the temperature boundary condition. It is therefore instructive to state different possible physical situations corresponding to the values of r with special reference to the heating or cooling of the bounding plates. computations of the solutions are performed for different values of bouyancy distribution parameter (r = 1, 2, 3, 4), (K = 0.2, 0.4, 0.6, 0.8) and (M = 1, 2, 3, 4) and for fixed reactant consumption parameter (λ) and activation energy parameter (ε).

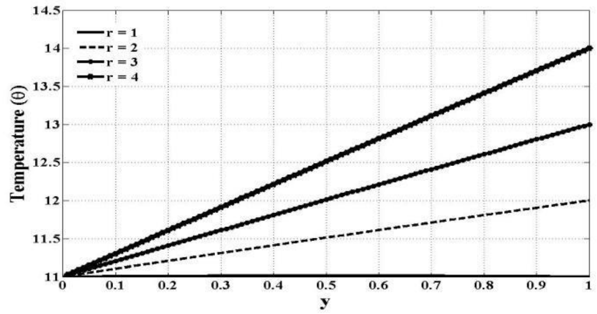

Figure 1 Effects of r on Temperature Profile

Figure (1) shows the temperature profiles for different values of r. It is reflected that the temperature increases with increase in r. The deviation of this symmetric behavior occurs more significantly as r increases, which correspands to the temperature increase at y = 1**.**

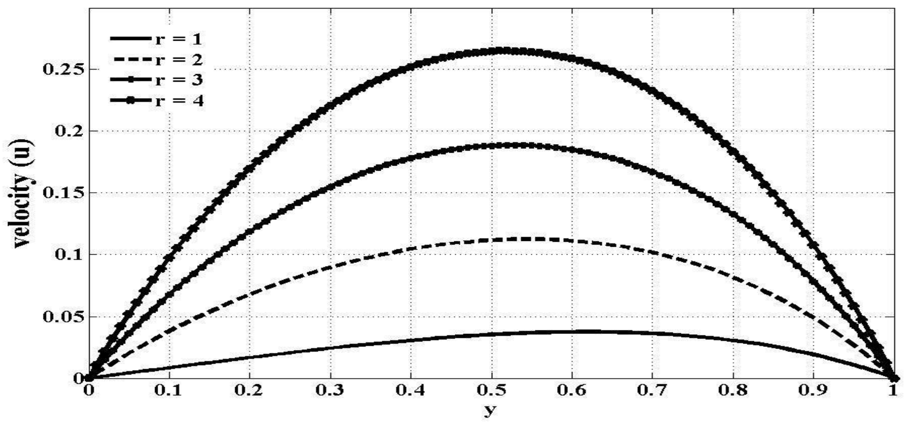

Figure 2 Effects of r on Velocity Profile

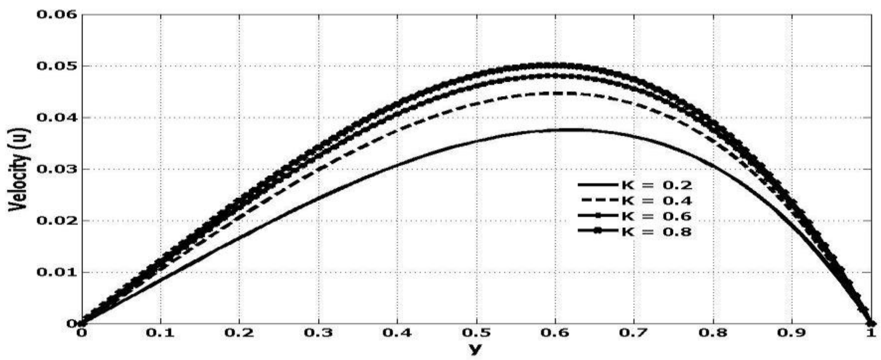

Figure 3 Effects of K on Velocity Profile

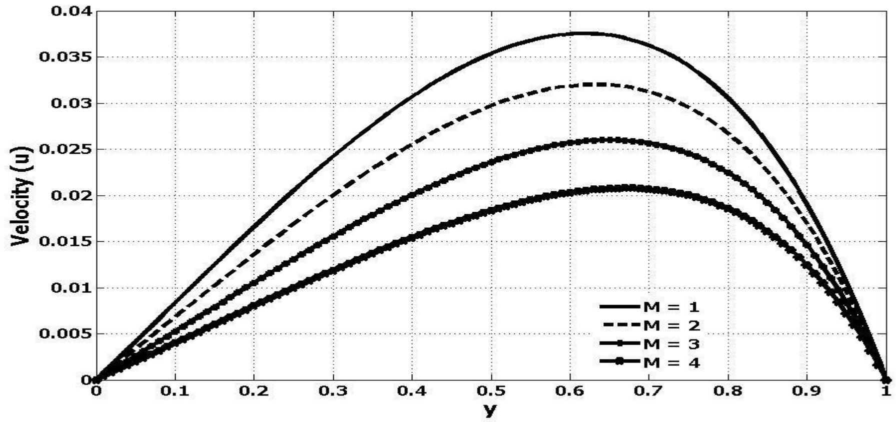

Figure 4 Effects of M on Velocity Profile

Figures (2) - (4) show the velocity profiles for different values of r, K and M. In Figure (2) it is seen that as r increases, the velocity increases. From Figure (3) it is observed that as K increases, the velocity increases. It is noted that the porosity parameter plays a major role in controlling the velocity. From Figure (4) it is obseved that as magnetic parameter increases, the velocity decreases. This indicates that the presence of magnetic field induces a Lorenz force which acts against the flow.

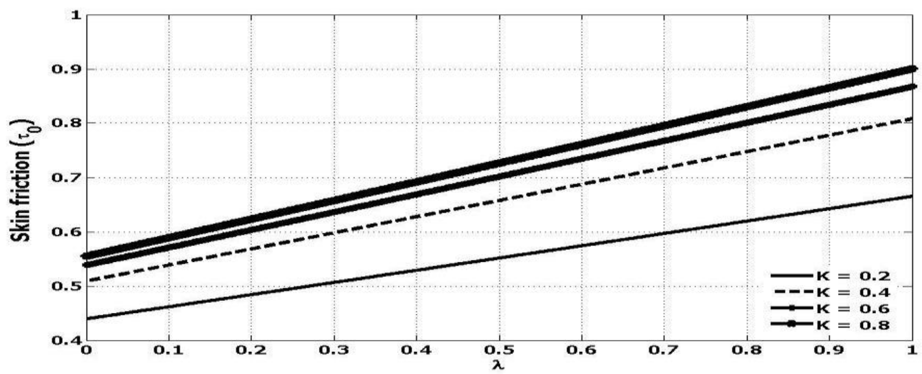

Figure 5 Effects of K on Skin Friction Profile at y = 0

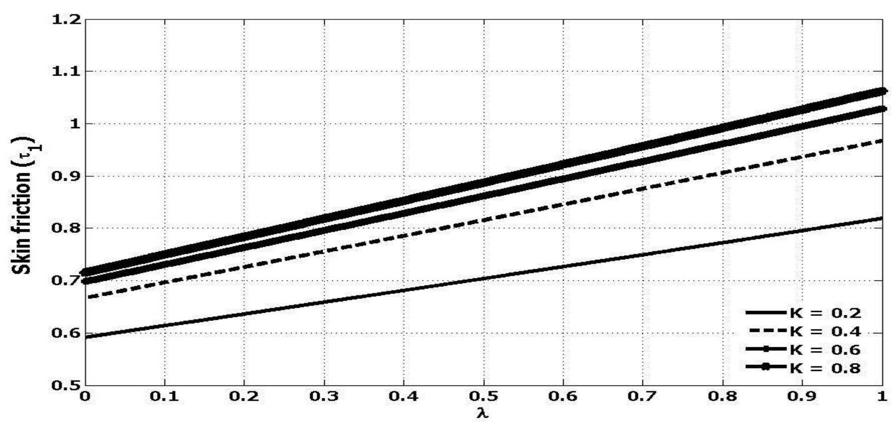

Figure 6 Effects of K on Skin Friction Profile at y = 1

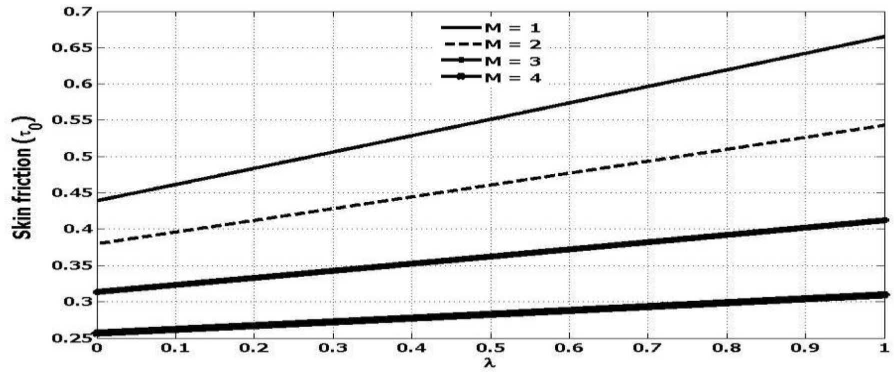

Figure 7 Effects of M on Skin Friction Profile at y = 0

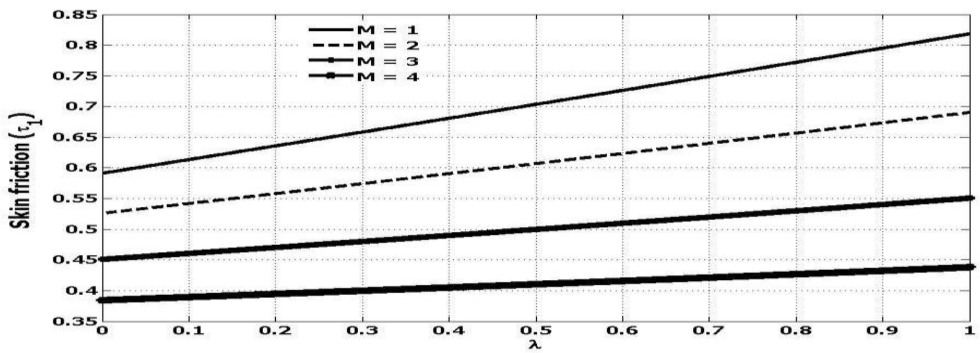

Figure 8 Effects of M on Skin Friction Profile at y = 1

The skin friction profiles for the different values of K and M at y = 0 and y = 1 are shown in Figures (5) - (8). From Figures (5) and (6) it is seen that as K increases, the skin friction increases. Similarly, from Figures (7) and (8), it is observed that as M increases, the skin friction decreases.

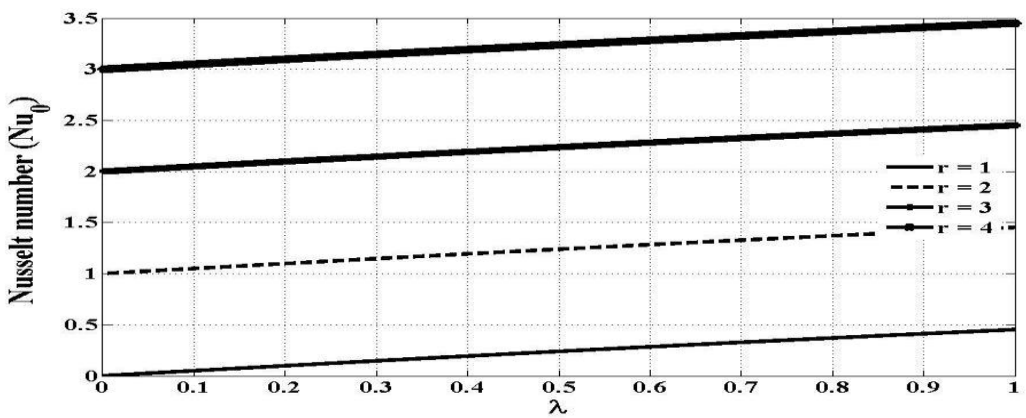

Figure 9 Effects of r on Nusselt Number Profile at y=0

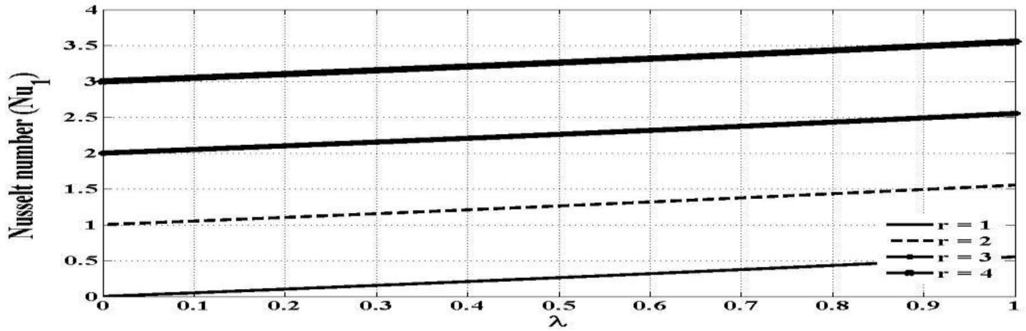

Figure 10 Effects of r on Nusselt Number Profile at y=0

Figures (9) and (10) show the variation of r on the rate of heat transfer. These two figures reveal that there is a monotonic increase in the rate of heat transfe at y = 0 and y =1.

CONCLUSION

The governing equations are approximated to a system of non-linear ordinary differential equations. The velocity and temperature profiles were obtained analytically using perturbation techniques and the expression for the skin-friction and the Nusselt number were obtained. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The results show that quantities of physical interest have significantly influenced the fluid flow formation under consideration. The velocity increases with increasing values of λ and K, The velocity decreases with the increase of the parameter M and The temperature increases with increasing values of λ and r.

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